AP Calculus AB + AP Physics 1
Complete Formula Sheet

Full formula addendum with true superscript notation: x^2, x^3, x^n, r^2, v^2, $a^2 + b^2 = c^2$. No caret notation is used, so multiplication and exponents are visually different.

How to use this sheet: read the formula, understand when to use it, then memorise the memory key. For calculation practice, always write the known values, choose the correct formula, substitute carefully, and check units.

Area	What to memorise first
Limits	Direct substitution, one-sided limits, important limits, continuity conditions
Derivatives	Derivative definition, power/product/quotient/chain rules, common trig/log/exponential derivatives
Integrals	Power rule, FTC, u-substitution, area, volume, average value, accumulation
Applications	motion, tangent/normal lines, extrema, concavity, optimisation, differential equations
Physics	kinematics, forces, work-energy, momentum, rotation, oscillations, fluids
Core math	algebra, exponents, logs, geometry, trigonometry, coordinate formulas

A. AP Calculus AB - Limits and Continuity

Start with limits. Many AP questions test whether you understand approaching a value, continuity, and behavior near holes/asymptotes.

Topic	Formula	When / Meaning	Memory Key
Limit sum law	lim(f + g) = lim f + lim g	A limit can be split across addition when both limits exist.	Separate easy parts first.
Limit product law	lim(fg) = (lim f)(lim g)	The limit of a product is the product of the limits.	Multiply after taking each limit.
Limit quotient law	lim(f/g) = (lim f)/(lim g), if lim g $\neq$ 0	The denominator limit cannot be zero.	Check denominator first.
Limit power law	$\lim(f(x))^n = (\lim f(x))^n$	Powers can pass through limits when the base limit exists.	Take the limit, then raise to the power.
Direct substitution	If f is continuous at c, $\lim_{x\to c} f(x) = f(c)$	Plug in c when the function is continuous there.	Substitute first if allowed.
Continuity at a point	f(c) exists, $\lim_{x\to c} f(x)$ exists, and $\lim_{x\to c} f(x) = f(c)$	All three conditions must be true.	Defined, limit exists, values match.
Squeeze theorem	If $g(x) \leq f(x) \leq h(x)$ and lim g = lim h = L, then lim f = L	A trapped function has the same limit as both bounds.	Same left and right walls force the middle.
Important trigonometric limit	$\lim_{x\to 0} \sin x / x = 1$	This is a core AP Calculus limit.	Sin over same angle approaches 1.
Important cosine limit	$\lim_{x\to 0} (1 - \cos x) / x = 0$	The numerator becomes small faster than x.	Cos gap over x goes to 0.
Exponential limit	$\lim_{x\to 0} (e^x - 1) / x = 1$	This supports the derivative of e^x.	e^x has slope 1 at 0.
Logarithmic limit	$\lim_{x\to 0} \ln(1 + x) / x = 1$	This supports the derivative of ln x.	Natural log grows like x near 0.
End behavior of rational functions	Compare highest powers of x in numerator and denominator	Highest degree terms control horizontal behavior.	Largest power wins.
Vertical asymptote check	Denominator = 0 and numerator $\neq$ 0	A rational function may blow up near zeros of the denominator.	Zero bottom, nonzero top.

B. AP Calculus AB - Derivative Rules

Memorise these until you can recognise the rule immediately. Derivatives measure slope, rate, and change.

Topic	Formula	When / Meaning	Memory Key
Derivative definition at a point	$f'(a) = \lim_{h\to0} [f(a+h) - f(a)] / h$	Instantaneous rate of change at x = a.	Slope from a shrinking secant line.
Derivative function definition	$f'(x) = \lim_{h\to0} [f(x+h) - f(x)] / h$	Turns the limit process into a formula for slope.	Difference quotient becomes derivative.
Constant rule	$d/dx[c] = 0$	A constant has no rate of change.	Flat line, zero slope.
Power rule	$d/dx[x^n] = n x^{n-1}$	Bring the exponent down and subtract 1 from the exponent.	Power down, exponent down.
Constant multiple rule	$d/dx[c f(x)] = c f'(x)$	Constants stay outside differentiation.	Leave the multiplier alone.
Sum and difference rule	$d/dx[f \pm g] = f' \pm g'$	Differentiate each term separately.	Term by term.
Product rule	$(fg)' = f'g + fg'$	Use when two variable expressions multiply.	Derivative first times second plus first times derivative second.
Quotient rule	$(f/g)' = (f'g - fg') / g^2$	Use when one function is divided by another.	Low d-high minus high d-low over low squared.
Chain rule	$d/dx[f(g(x))] = f'(g(x))g'(x)$	Differentiate the outside, keep inside, then multiply by inside derivative.	Outside, inside, multiply.
Derivative of sin x	$d/dx[\sin x] = \cos x$	Sine differentiates to cosine.	Sin goes to cos.
Derivative of cos x	$d/dx[\cos x] = -\sin x$	Cosine differentiates to negative sine.	Cos goes to negative sin.
Derivative of tan x	$d/dx[\tan x] = \sec^2 x$	Tangent differentiates to secant squared.	Tan becomes $\sec^2$.
Derivative of sec x	$d/dx[\sec x] = \sec x \tan x$	Common AP trig derivative.	Sec carries tan.
Derivative of csc x	$d/dx[\csc x] = -\csc x \cot x$	Common AP trig derivative.	Csc is negative with cot.
Derivative of cot x	$d/dx[\cot x] = -\csc^2 x$	Common AP trig derivative.	Cot becomes negative $\csc^2$.
Derivative of e^x	$d/dx[e^x] = e^x$	e^x is its own derivative.	Same function.
Derivative of a^x	$d/dx[a^x] = a^x \ln a$	For a constant base a > 0.	Keep a^x and multiply ln a.
Derivative of ln x	$d/dx[\ln x] = 1/x$	Natural log derivative.	Log becomes reciprocal.
Derivative of $\log_a$ x	$d/dx[\log_a x] = 1/(x \ln a)$	For logarithm with base a.	Natural log adjustment.
Derivative of arcsin x	$d/dx[\arcsin x] = 1/\sqrt{1 - x^2}$	Inverse sine derivative.	Positive reciprocal root.
Derivative of arccos x	$d/dx[\arccos x] = -1/\sqrt{1 - x^2}$	Inverse cosine derivative.	Negative reciprocal root.
Derivative of arctan x	$d/dx[\arctan x] = 1/(1 + x^2)$	Inverse tangent derivative.	One over one plus square.
Implicit differentiation	Differentiate both sides with respect to x and multiply y-terms by dy/dx	Used when y is not isolated.	Every y needs dy/dx.
Inverse derivative	$(f^{-1})'(a) = 1 / f'(f^{-1}(a))$	Slope of inverse is reciprocal slope of original at the matching point.	Inverse slope flips.

C. AP Calculus AB - Derivative Applications

Use derivatives to explain motion, optimisation, tangent lines, increasing/decreasing behavior, and concavity.

Topic	Formula	When / Meaning	Memory Key
Tangent line	$y - y_1 = m(x - x_1)$, where $m = f'(x_1)$	Use derivative as slope at a point.	Derivative gives tangent slope.
Normal line slope	m_normal = -1 / m_tangent	Normal line is perpendicular to tangent.	Perpendicular slopes multiply to -1.
Position, velocity, acceleration	$v(t) = s'(t)$, $a(t) = v'(t) = s''(t)$	Derivatives connect motion quantities.	Position → velocity → acceleration.
Speed	speed = $\|v(t)\|$	Speed is magnitude of velocity.	Remove direction with absolute value.
Increasing function	$f'(x) > 0$	Positive derivative means function rises.	Positive slope rises.
Decreasing function	$f'(x) < 0$	Negative derivative means function falls.	Negative slope falls.
Critical point	$f'(x) = 0$ or $f'(x)$ undefined	Critical points are candidates for extrema.	Zero or broken derivative.
First derivative test	f' changes + to - gives local maximum; - to + gives local minimum	Sign change of derivative identifies local extrema.	Up then down max; down then up min.
Concavity up	$f''(x) > 0$	Derivative is increasing; graph bends upward.	Smile shape.
Concavity down	$f''(x) < 0$	Derivative is decreasing; graph bends downward.	Frown shape.
Inflection point	Concavity changes sign	Second derivative sign change indicates possible inflection.	Concavity must switch.
Second derivative test	If $f'(c)=0$ and $f''(c)>0$, local min; if $f''(c)<0$, local max	Uses concavity at a critical point.	Positive second derivative makes a bowl.
Mean Value Theorem	$f'(c) = [f(b) - f(a)] / (b - a)$	Some tangent slope equals average slope if conditions hold.	Instant slope equals average slope somewhere.
Rolle’s theorem	If $f(a)=f(b)$, then $f'(c)=0$ for some c	Special case of MVT with equal endpoint values.	Same height means flat tangent somewhere.
Linearization	$L(x) = f(a) + f'(a)(x - a)$	Tangent line approximation near x = a.	Start value plus slope times change.
Differential approximation	$dy \approx f'(x)\,dx$	Small output change is derivative times input change.	Change ≈ slope × input change.

D. AP Calculus AB - Integral Rules and Applications

Integrals measure accumulation, net change, area, and volume. The Fundamental Theorem of Calculus connects derivatives and integrals.

Topic	Formula	When / Meaning	Memory Key
Antiderivative power rule	$\int x^n\,dx = x^{n+1}/(n+1) + C,\ n \neq -1$	Add 1 to exponent and divide by new exponent.	Power up, divide by new power.
Integral of 1/x	$\int 1/x\,dx = \ln\lvert x\rvert + C$	Special antiderivative for reciprocal.	Reciprocal integrates to log.
Integral of e^x	$\int e^x\,dx = e^x + C$	e^x remains itself under integration.	Same function.
Integral of a^x	$\int a^x\,dx = a^x/\ln a + C$	Exponential base a needs ln a adjustment.	Divide by ln base.
Integral of cos x	$\int \cos x\,dx = \sin x + C$	Cosine integrates to sine.	Cos goes to sin.
Integral of sin x	$\int \sin x\,dx = -\cos x + C$	Sine integrates to negative cosine.	Sin goes to negative cos.
Integral of $\sec^2$ x	$\int \sec^2 x\,dx = \tan x + C$	Derivative of tangent is secant squared.	Sec^2 returns tan.
Integral of $\csc^2$ x	$\int \csc^2 x\,dx = -\cot x + C$	Derivative of cotangent is negative $\csc^2$.	Csc^2 returns negative cot.
Integral of sec x tan x	$\int \sec x \tan x\,dx = \sec x + C$	Derivative of secant is secant tangent.	Sec tan returns sec.
Integral of csc x cot x	$\int \csc x \cot x\,dx = -\csc x + C$	Derivative of cosecant is negative cosecant cotangent.	Csc cot returns negative csc.
u-substitution	If $u = g(x)$, then $\int f(g(x))g'(x)\,dx = \int f(u)\,du$	Reverse chain rule.	Find inside and its derivative.
Definite integral property	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$	Switching bounds changes sign.	Reverse bounds reverse sign.
Zero-width integral	$\int_a^a f(x)\,dx = 0$	No interval means no accumulated area.	Same bounds, zero area.
Additive interval property	$\int_a^b f\,dx + \int^{bc} f\,dx = \int_a^c f\,dx$	Adjacent integrals combine.	Add intervals end to end.
Fundamental Theorem Part 1	If $F(x)=\int_a^x f(t)\,dt$, then $F'(x)=f(x)$	Derivative of accumulation gives the original rate.	Derivative cancels accumulation.
Fundamental Theorem Part 2	$\int_a^b f(x)\,dx = F(b) - F(a)$	Evaluate antiderivative at upper minus lower bound.	Top minus bottom.
Average value of a function	$f\_avg = 1/(b-a) \int_a^b f(x)\,dx$	Mean output value over an interval.	Total accumulation divided by length.
Net change theorem	$F(b) - F(a) = \int_a^b F'(x)\,dx$	Accumulated rate gives total change.	Integrate rate to get change.
Area between curves	$\text{Area} = \int_a^b [\text{top} - \text{bottom}]\,dx$	Subtract lower curve from upper curve.	Top minus bottom.
Volume by disks	$V = \pi\int_a^b [R(x)]^2\,dx$	Cross-sectional area is a circle.	π radius squared integrated.

Topic	Formula	When / Meaning	Memory Key
Volume by washers	$V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)\,dx$	Outer disk minus inner hole.	Big radius squared minus small radius squared.
Volume by known cross sections	$V = \int_a^b A(x)\,dx$	Integrate cross-sectional area.	Area slices add volume.
Separable differential equation	$dy/dx = g(x)h(y) \rightarrow dy/h(y) = g(x)\,dx$	Separate x and y before integrating.	Move y with dy, x with dx.
Exponential growth or decay	$dy/dt = ky \rightarrow y = Ce^{kt}$	Rate proportional to amount.	Proportional change gives exponential model.
Logistic model	$dy/dt = ky(1 - y/L)$	Growth slows toward carrying capacity L.	Fast first, then levels off.
Trapezoidal rule	Approx. area = $\Delta x/2\,[y_0 + 2y_1 + 2y_2 + \ldots + y_n]$	Approximates area using trapezoids.	Ends once, middle twice.
Left Riemann sum	$L_n = \Sigma f(\text{left endpoint})\,\Delta x$	Uses left endpoint heights.	Left side rectangles.
Right Riemann sum	$R_n = \Sigma f(\text{right endpoint})\,\Delta x$	Uses right endpoint heights.	Right side rectangles.
Midpoint Riemann sum	$M_n = \Sigma f(\text{midpoint})\,\Delta x$	Uses midpoint heights.	Middle gives better balance.

E. Core Mathematics Support - Algebra, Trigonometry, Geometry, Logs

These formulas support both AP Calculus AB and AP Physics 1. The notation uses real superscripts, for example x^2 and r^2.

Topic	Formula	When / Meaning	Memory Key
Quadratic formula	$x = (-b \pm \sqrt{b^2 - 4ac}) / (2a)$	Solves $ax^2 + bx + c = 0$.	Discriminant decides number of roots.
Discriminant	$D = b^2 - 4ac$	$D > 0$ two real roots, $D = 0$ one repeated root, $D < 0$ no real roots.	Check D before solving.
Slope	$m = (y_2 - y_1)/(x_2 - x_1)$	Rate of change between two points.	Rise over run.
Line slope-intercept	$y = mx + b$	m is slope and b is y-intercept.	Slope plus starting height.
Point-slope form	$y - y_1 = m(x - x_1)$	Line through a point with slope m.	Point plus slope.
Distance formula	$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$	Distance between two coordinate points.	Pythagorean theorem on coordinates.
Midpoint formula	$M = ((x_1+x_2)/2, (y_1+y_2)/2)$	Average the x-values and y-values.	Average coordinates.
Circle equation	$(x-h)^2 + (y-k)^2 = r^2$	Circle with center (h,k) and radius r.	Distance from center equals radius.
Pythagorean theorem	$a^2 + b^2 = c^2$	Right triangle side relationship.	Leg squared plus leg squared equals hypotenuse squared.
45-45-90 triangle	$x : x : x\sqrt{2}$	Special right triangle side ratio.	Equal legs, hypotenuse has $\sqrt{2}$.
30-60-90 triangle	$x : x\sqrt{3} : 2x$	Special right triangle side ratio.	Short leg, long leg, hypotenuse.
Sine ratio	$\sin\theta$ = opposite / hypotenuse	Trigonometric ratio.	SOH.
Cosine ratio	$\cos\theta$ = adjacent / hypotenuse	Trigonometric ratio.	CAH.
Tangent ratio	$\tan\theta$ = opposite / adjacent	Trigonometric ratio.	TOA.
Pythagorean trig identity	$\sin^2\theta + \cos^2\theta = 1$	Fundamental trig identity.	Squares of sine and cosine add to one.
Tangent identity	$\tan\theta = \sin\theta / \cos\theta$	Connects tangent with sine and cosine.	Tan is sine over cosine.
Radians and degrees	π radians = 180°	Convert between angular units.	Half-turn equals π radians.
Unit circle coordinates	$(\cos\theta, \sin\theta)$	Point on unit circle at angle θ.	x is cosine, y is sine.
Area of triangle	$A = 1/2\ bh$	Base times height divided by 2.	Half a rectangle.
Area of circle	$A = \pi r^2$	Area inside a circle.	Pi times radius squared.
Circumference	$C = 2\pi r = \pi d$	Distance around a circle.	Pi times diameter.
Arc length	$s = r\theta$	θ must be in radians.	Radius times angle.

Topic	Formula	When / Meaning	Memory Key
Sector area	$A = 1/2\ r^2\theta$	θ must be in radians.	Half radius squared times angle.
Exponent product rule	$a^m a^n = a^{m+n}$	Same base multiplication adds exponents.	Same base, add powers.
Exponent quotient rule	$a^m/a^n = a^{m-n}$	Same base division subtracts exponents.	Same base, subtract powers.
Power of a power	$(a^m)^n = a^{mn}$	Multiply exponents.	Power times power.
Negative exponent	$a^{-n} = 1/a^n$	Negative exponent means reciprocal.	Move to denominator.
Zero exponent	$a^0 = 1, a \neq 0$	Any nonzero base to zero power equals one.	Zero power gives one.
Log product rule	$\ln(ab) = \ln a + \ln b$	Log of product becomes sum.	Multiply inside, add outside.
Log quotient rule	$\ln(a/b) = \ln a - \ln b$	Log of quotient becomes difference.	Divide inside, subtract outside.
Log power rule	$\ln(a^n) = n \ln a$	Exponent moves in front.	Power drops down.

F. AP Physics 1 - Complete Formula Set

Use these with diagrams and units. AP Physics 1 problems usually require reasoning, not only substitution.

Topic	Formula	When / Meaning	Memory Key
Average velocity	$v_{avg} = \Delta x / \Delta t$	Change in position divided by change in time.	Displacement over time.
Average acceleration	$a_{avg} = \Delta v / \Delta t$	Change in velocity divided by change in time.	Velocity change over time.
Constant acceleration velocity	$v = v_0 + at$	Velocity after time t under constant acceleration.	Initial plus acceleration time.
Constant acceleration position	$x = x_0 + v_0 t + 1/2\ at^2$	Position under constant acceleration.	Initial position plus motion terms.
Velocity-displacement relation	$v^2 = v_0^2 + 2a(x - x_0)$	Connects velocity and displacement without time.	Use when time is missing.
Average velocity constant acceleration	$v_{avg} = (v_0 + v)/2$	Average of initial and final velocities.	Only for constant acceleration.
Projectile horizontal motion	$x = x_0 + v_{0x} t$	Horizontal acceleration is usually zero.	Horizontal velocity stays constant.
Projectile vertical motion	$y = y_0 + v_{0y} t - 1/2\ gt^2$	Vertical motion has gravitational acceleration downward.	Treat vertical separately.
Vector magnitude	$\lvert A \rvert = \sqrt{(A_x^2 + A_y^2)}$	Magnitude from perpendicular components.	Pythagorean vector length.
Vector components	$A_x = A \cos\theta$, $A_y = A \sin\theta$	Break a vector into horizontal and vertical parts.	Cos with x, sin with y.
Newton's second law	$\Sigma F = ma$	Net force causes acceleration.	Net force, not one force.
Weight	$F_g = mg$	Gravitational force near Earth.	Mass times gravity.
Static friction maximum	$f_s \le \mu_s N$	Static friction adjusts up to a maximum.	Static friction is not always $\mu_s N$.
Kinetic friction	$f_k = \mu_k N$	Friction while sliding.	Sliding friction equals coefficient times normal force.
Spring force	$F_s = -kx$	Restoring force points opposite displacement.	Spring pulls back.
Centripetal acceleration	$a_c = v^2/r$	Acceleration toward center of circular path.	Speed squared over radius.
Centripetal force	$\Sigma F_c = mv^2/r$	Net inward force for circular motion.	Not a new force; net inward force.
Work by constant force	$W = Fd \cos\theta$	Only the force component along displacement does work.	Parallel component matters.
Kinetic energy	$K = 1/2\ mv^2$	Energy of motion.	Mass times speed squared, half.
Gravitational potential energy	$U_g = mgh$	Energy due to height near Earth.	Higher means more gravitational energy.
Spring potential energy	$U_s = 1/2\ kx^2$	Energy stored in stretched or compressed spring.	Spring energy grows with x^2.
Work-energy theorem	$W_{net} = \Delta K$	Net work changes kinetic energy.	Net work becomes motion energy.

Topic	Formula	When / Meaning	Memory Key
Conservation of mechanical energy	$K_i + U_i = K_f + U_f$	Use when nonconservative work is zero.	Energy changes form, total stays same.
Power	$P = W/t = Fv$	Rate of doing work or transferring energy.	Energy per time.
Momentum	$p = mv$	Mass times velocity.	Motion quantity with direction.
Impulse	$J = F_{avg} \Delta t = \Delta p$	Impulse changes momentum.	Force over time changes momentum.
Momentum conservation	$\Sigma p_i = \Sigma p_f$	Use when external impulse is negligible.	Total momentum stays fixed.
Center of mass	$x_{cm} = \Sigma m_i x_i / \Sigma m_i$	Weighted average position.	Heavier masses count more.
Torque	$\tau = rF \sin\theta$	Rotational effect of force.	Lever arm times perpendicular force.
Rotational equilibrium	$\Sigma\tau = 0$ and $\Sigma F = 0$	Object is not accelerating linearly or rotationally.	No net force and no net torque.

Topic	Formula	When / Meaning	Memory Key
Angular velocity	$\omega = \Delta\theta/\Delta t$	Rate of angular displacement.	Angle per time.
Angular acceleration	$\alpha = \Delta\omega/\Delta t$	Rate of angular velocity change.	Angular velocity change over time.
Rotational kinematics velocity	$\omega = \omega_0 + \alpha t$	Angular analog of linear velocity equation.	Same pattern as $v = v_0 + at$.
Rotational kinematics position	$\theta = \theta_0 + \omega_0 t + 1/2\ \alpha t^2$	Angular position under constant angular acceleration.	Same pattern as x equation.
Rotational speed relation	$v = r\omega$	Linear speed at radius r.	Farther out moves faster.
Tangential acceleration	$a_t = r\alpha$	Linear acceleration along tangent.	Radius times angular acceleration.
Moment of inertia point mass	$I = mr^2$	Rotational inertia of point mass.	Mass farther away resists more.
Rotational Newton's second law	$\Sigma\tau = I\alpha$	Net torque causes angular acceleration.	Torque is rotational force.
Rotational kinetic energy	$K_{rot} = 1/2\ I\omega^2$	Energy of rotation.	Rotational version of kinetic energy.
Angular momentum	$L = I\omega$	Rotational momentum.	Rotational inertia times angular velocity.
Angular momentum conservation	$L_i = L_f$	Use when net external torque is zero.	No outside torque, angular momentum stays.
Rolling without slipping	$v_{cm} = R\omega$	Linear and angular motion linked.	No slipping means matching motion.
Simple harmonic motion period spring	$T = 2\pi\sqrt{(m/k)}$	Period of mass-spring oscillator.	More mass slower; stiffer spring faster.
Simple pendulum period	$T = 2\pi\sqrt{(L/g)}$	Small-angle pendulum period.	Longer pendulum slower.

Topic	Formula	When / Meaning	Memory Key
Oscillation frequency	$f = 1/T$	Frequency is cycles per second.	Frequency and period are reciprocals.
Wave speed	$v = f\lambda$	Wave speed equals frequency times wavelength.	Speed equals cycles times length per cycle.
Density	$\rho = m/V$	Mass per unit volume.	More mass in same volume means greater density.
Pressure	$P = F/A$	Force per unit area.	Same force on smaller area means higher pressure.
Gauge pressure in fluid	$P = P_0 + \rho gh$	Pressure increases with depth.	Deeper fluid gives more pressure.
Buoyant force	$F_b = \rho_{fluid} V_{displaced}\ g$	Upward force equals weight of displaced fluid.	Object floats by displacing fluid.
Continuity equation	$A_1v_1 = A_2v_2$	Fluid flow rate stays constant in steady incompressible flow.	Narrow pipe, faster fluid.
Bernoulli's equation	$P + 1/2\ \rho v^2 + \rho gy = \text{constant}$	Energy conservation for flowing fluid.	Pressure, motion, and height trade off.
Universal gravitation	$F_g = Gm_1m_2/r^2$	Attractive force between two masses.	More mass more force; more distance less force.
Gravitational field	$g = GM/r^2$	Gravitational acceleration due to a mass M.	Field from a central mass.
Hooke's law energy connection	$F = kx$ and $U_s = 1/2kx^2$	Force is linear in stretch; energy is quadratic.	Force straight line, energy curve.
Unit conversion acceleration	$1\ m/s^2$ = 1 meter per second per second	Acceleration changes velocity every second.	Velocity change rate.

G. Exam Memory Strategy

Use this page before every practice session to decide which tool to apply.

Topic	Formula	When / Meaning	Memory Key
Calculus priority 1	limits → derivatives → integrals → applications	Study order for AP Calculus AB.	Concepts build in this order.
Calculus priority 2	f' = slope/rate; f'' = concavity/change of rate	Interpret derivatives in words.	First derivative slope, second derivative bend.
Calculus priority 3	$\int$ rate = total change	Core meaning of definite integrals.	Accumulation converts rate to amount.
Physics priority 1	draw diagram → choose system → write knowns → choose formula → solve units	Process for AP Physics 1 problems.	Diagram first, equation second.
Physics priority 2	forces cause acceleration; energy tracks work; momentum tracks interactions	Pick the correct physics tool.	Force, energy, momentum are different lenses.
Physics priority 3	always check units	Units often reveal mistakes.	Wrong unit usually means wrong method.

Official Reference Notes

This formula sheet is an original study aid. It is based on AP Calculus AB and AP Physics 1 topics and should be used with official College Board AP course/exam descriptions, AP Physics 1 equation/reference information, and official free-response questions. Some formulas are provided on official reference sheets, while many calculus derivative/integral rules must be memorised and practised.

Reference links: AP Calculus AB exam page, AP Calculus AB/BC Course and Exam Description, AP Physics 1 exam page, AP Physics 1 Course and Exam Description, and AP Physics 1 Exam Reference Information.

Detailed Step-by-Step Solution Guide
AP Calculus AB + AP Physics 1

This addendum keeps the original workbook unchanged and adds clearer solution methods. Use it before each drill section: read the question, identify the type, follow the checklist, then compare your work with the direct answer in the workbook.

Universal solving routine

1. Identify the topic and the target value.
2. Write known values and units.
3. Choose the rule/formula from the formula sheet.
4. Substitute carefully using parentheses, especially for negative values.
5. Simplify one line at a time.
6. Check reasonableness: sign, unit, graph shape, and magnitude.

A. AP Calculus AB - Detailed Solution Methods

Limits and continuity

Type	How to solve	Worked mini-example
Direct substitution	Try f(c). If it gives a real number and the function is continuous, that is the limit.	$\lim_{x\to 2} (x^2 + 3x) = 2^2 + 3(2) = 4 + 6 = 10$.
Factor and cancel	If substitution gives 0/0, factor numerator/denominator, cancel the common factor, then substitute again.	$\lim_{x\to 3} (x^2 - 9)/(x - 3) = \lim_{x\to 3} [(x - 3)(x + 3)]/(x - 3) = 6$.
Rationalize	If a square root creates 0/0, multiply by the conjugate.	$\lim_{x\to 0} (\sqrt{x+4}-2)/x \times (\sqrt{x+4}+2)/(\sqrt{x+4}+2) = 1/(\sqrt{x+4}+2) \to 1/4$.
One-sided limits	Check left and right sides separately. The two-sided limit exists only if both sides match.	For $f(x)=\|x\|/x$ at $x=0$: left = -1, right = 1, so the two-sided limit does not exist.

Derivative rules

Type	How to solve	Worked mini-example
Power rule	For ax^n, multiply coefficient by n and subtract 1 from the exponent.	$d/dx(6x^5 + 2x) = 30x^4 + 2$.
Product rule	Use $f'g + fg'$ when two variable expressions multiply.	$d/dx[(x^2+1)\sin x] = 2x \sin x + (x^2+1)\cos x$.
Quotient rule	Use (low × derivative high - high × derivative low) / low².	$d/dx[(x^2+1)/(x-1)] = [(x-1)(2x) - (x^2+1)(1)]/(x-1)^2$.
Chain rule	Differentiate the outside, keep the inside, multiply by inside derivative.	$d/dx[(3x^2+1)^5] = 5(3x^2+1)^4(6x)$.
Implicit differentiation	Differentiate both sides. Every derivative of y must include dy/dx.	$x^2 + y^2 = 25 \to 2x + 2y(dy/dx)=0 \to dy/dx = -x/y$.

Derivative applications

Type	How to solve	Worked mini-example
Tangent line	Find slope $m=f'(a)$, find point $(a,f(a))$, then use $y-y_1=m(x-x_1)$.	If $f(x)=x^2$ at $x=3$, slope=6 and point=(3,9), so $y-9=6(x-3)$.
Increasing/decreasing	Solve $f'(x)>0$ or $f'(x)<0$, then describe intervals.	If $f'(x)=(x-1)(x+2)$, test intervals around -2 and 1.
Concavity	Use $f''(x)$. Positive means concave up; negative means concave down.	If f'' changes sign at c, c is an inflection point candidate.
Optimization	Define variable, build equation, reduce to one variable, differentiate, set derivative to 0, test endpoints if needed.	Max area problems often use area formula plus a constraint equation.
Related rates	Draw, write equation, differentiate with respect to time, substitute values at the end.	Do not substitute too early because variables are changing.

Integrals and accumulation

Type	How to solve	Worked mini-example
Power integration	Increase the exponent by 1, divide by the new exponent, add C for indefinite integrals.	$\int 7x^2\, dx = 7x^3/3 + C$.
u-substitution	Choose the inside expression as u. Its derivative must appear or be adjusted.	$\int 2x(x^2+1)^5\, dx$: let $u=x^2+1$, $du=2x\, dx$, so $\int u^5 du = u^6/6 + C$.
FTC Part 2	For definite integrals, find antiderivative F and calculate F(b)-F(a).	$\int_0^2 3x^2\, dx = [x^3]_0^2 = 8$.
Area between curves	Use ∫(top-bottom) dx or ∫(right-left) dy.	Always sketch or compare values to identify top and bottom.

Type	How to solve	Worked mini-example
Volume	Disk: $\pi\int R^2\,dx$. Washer: $\pi\int (R^2-r^2)\,dx$. Cross-section: $\int A(x)dx$.	Square radius first, then integrate.
Differential equations	Separate variables if possible, integrate both sides, solve for C using initial condition.	$dy/dx=ky \rightarrow dy/y = k\,dx \rightarrow \ln\|y\| = kx + C \rightarrow y=Ce^{kx}$.

AP Calculus AB - FRQ Writing Template

For free-response work, do not only write the final number. Show the rule, substitution, and interpretation. A strong answer usually has four lines: formula, substitution, simplification, final statement with units or meaning.

FRQ demand	What to write	Common mistake to avoid
Find a limit	Show algebraic simplification or one-sided comparison before final value.	Writing only the final value after a 0/0 form.
Find derivative at a point	State derivative formula first, then substitute the point.	Forgetting chain rule or product rule.
Justify maximum/minimum	Use sign chart, first derivative test, second derivative test, or endpoint comparison.	Saying “it is maximum” without justification.
Interpret integral	Explain accumulated change or area with units.	Calling every integral “area” even when it represents total change.
Differential equation	Separate variables and show initial condition step.	Forgetting +C or using initial value incorrectly.

B. AP Physics 1 - Detailed Solution Methods

General physics workflow

Type	How to solve	Worked mini-example / warning
Units first	Convert to SI units before substituting: m, kg, s, N, J, W, Pa.	If distance is 30 cm, write 0.30 m before using formulas.
Diagram	Draw direction, forces, velocity, acceleration, axes, and given values.	A free-body diagram should show forces, not velocity vectors.
Choose system	For energy and momentum, decide what is inside the system.	If two carts collide, system = both carts for conservation of momentum.
Check sign	Pick positive direction and keep it consistent.	Upward can be positive, so gravitational acceleration is -g.

Kinematics and graphs

Type	How to solve	Worked mini-example / warning
Constant acceleration	Use $v=v_0+at$, $x=x_0+v_0t+\frac{1}{2}at^2$, or $v^2=v_0^2+2a\Delta x$ depending on missing variable.	If time is missing, use $v^2=v_0^2+2a\Delta x$.
Projectile motion	Split horizontal and vertical motion. Time is shared; horizontal acceleration is usually 0.	Find time from vertical equation, then use horizontal equation for range.
Graphs	Slope of x-t graph = velocity; slope of v-t graph = acceleration; area under v-t graph = displacement.	Do not confuse height of graph with slope.

Forces and circular motion

Type	How to solve	Worked mini-example / warning
Free-body diagrams	Draw only real forces: weight, normal, tension, friction, applied force, spring force.	Do not draw “ma” as a force.
Newton’s second law	Write $\Sigma F=ma$ along each axis separately.	For no vertical acceleration, $\Sigma F_y=0$.
Incline	Resolve weight into $mg \sin\theta$ down slope and $mg \cos\theta$ into surface.	Normal force on incline is usually $mg \cos\theta$, not mg.
Circular motion	Net inward force equals mv^2/r.	Centripetal force is the net inward force, not a new force.

Energy and momentum

Type	How to solve	Worked mini-example / warning
Work-energy	Use $W_{net}=\Delta K$ or $K_i+U_i+W_{external}=K_f+U_f$.	Use energy when forces change with position or path is easier.
Spring energy	$U_s=\frac{1}{2}kx^2$. The x is compression/stretch from equilibrium.	Do not use total length; use displacement from natural length.
Momentum	Use $p=mv$ and impulse $J=F\Delta t=\Delta p$.	Momentum is vector; direction matters.
Collisions	Momentum is conserved if external impulse is negligible. Kinetic energy only conserved in elastic collisions.	Do not conserve kinetic energy for inelastic collisions unless stated.

Rotation, oscillations, fluids

Type	How to solve	Worked mini-example / warning
Torque	$\tau=rF \sin\theta$. Choose pivot, assign clockwise/counterclockwise signs.	If line of action passes through pivot, torque is zero.
Rotational dynamics	Use $\tau_{net}=I\alpha$ and connect $a=\alpha r$ when rolling without slipping.	Use angular equivalents: force→torque, mass→moment of inertia, acceleration→angular acceleration.

Type	How to solve	Worked mini-example / warning
Simple harmonic motion	Look for restoring force proportional to displacement.	For a spring, $T=2\pi\sqrt{m/k}$.
Fluids	Pressure $p=F/A$, $p=p_0+\rho gh$, buoyant force=$\rho_{fluid} V_{displaced}\, g$.	Object floats when weight equals buoyant force.

AP Physics 1 - FRQ Answer Structure

FRQ type	Best answer structure	What earns clarity
Mathematical routine	State equation, substitute variables, solve symbolically if possible, then evaluate.	Units, algebra, and a final sentence.
Experimental design	State independent variable, dependent variable, controls, measurement tools, and expected relationship.	A clear repeatable procedure.
Graph/representation	Connect slope, area, intercept, or shape to physical meaning.	Explicitly say what slope/area represents.
Argument/justification	Use a law or principle, then connect it to the given situation.	Because + physics principle + evidence from problem.

Final readiness rule

You are ready for a timed mock when you can show setup lines without looking at the solution guide.

For each wrong answer, write: topic, mistake type, correct method, and one new similar practice question.

Expanded Worked Examples
AP Calculus AB + AP Physics 1

This expanded section gives full worked examples in the same style students should write during practice: known values, formula choice, substitution, calculation, and final interpretation.

Limit by factoring

Problem: Evaluate lim $x \to 3$ $(x^2 - 9)/(x - 3)$.

Step 1: Direct substitution gives 0/0, so it is indeterminate and needs algebra.

Step 2: Factor the numerator: $x^2-9=(x-3)(x+3)$.

Step 3: Cancel the common factor x-3 because x is approaching 3, not equal to 3.

Step 4: Substitute x=3 into x+3 to get 6.

Step 5: Final answer: 6.

Memory key: For 0/0, factor/cancel before substituting again.

Limit by rationalising

Problem: Evaluate lim $x \to 0$ $(\sqrt{(x+4)}-2)/x$.

Step 1: Substitution gives 0/0, so rationalise using the conjugate.

Step 2: Multiply top and bottom by $\sqrt{(x+4)}+2$.

Step 3: The numerator becomes (x+4)-4=x.

Step 4: Cancel x with the denominator.

Step 5: Evaluate $1/(\sqrt{(x+4)}+2)$ at x=0 to get 1/4.

Memory key: Roots with 0/0 often need conjugates.

Derivative with chain rule

Problem: Differentiate $y=(3x^2+1)^5$.

Step 1: Identify outside function u^5 and inside $u=3x^2+1$.

Step 2: Differentiate outside: $5u^4$.

Step 3: Differentiate inside: du/dx=6x.

Step 4: Multiply: $dy/dx=5(3x^2+1)^4(6x)=30x(3x^2+1)^4$.

Memory key: Outside, keep inside, multiply by inside derivative.

Tangent line

Problem: Find tangent line to $f(x)=x^2+2x$ at x=1.

Step 1: Find the point: $f(1)=1^2+2(1)=3$, so point is (1,3).

Step 2: Differentiate: $f'(x)=2x+2$.

Step 3: Find slope: $f'(1)=4$.

Step 4: Use point-slope form: y-3=4(x-1).

Step 5: Simplify if needed: y=4x-1.

Memory key: Derivative gives slope; function gives point.

Optimization

Problem: A rectangle has perimeter 40. Find maximum area.

Step 1: Let sides be x and y. Constraint: $2x+2y=40$, so $y=20-x$.

Step 2: Area $A=xy=x(20-x)=20x-x^2$.

Step 3: Differentiate: $A'=20-2x$.

Step 4: Set $A'=0$: $20-2x=0$, so $x=10$. Then $y=10$.

Step 5: Maximum area is 100 square units.

Memory key: Use constraint to reduce to one variable.

Definite integral

Problem: Evaluate $\int_0^2 (3x^2+4)\,dx$.

Step 1: Find antiderivative: $\int 3x^2dx=x^3$ and $\int 4dx=4x$.

Step 2: So $F(x)=x^3+4x$.

Step 3: Apply FTC: $F(2)-F(0)=(8+8)-0=16$.

Step 4: Final answer: 16.

Memory key: Top bound minus bottom bound.

Area between curves

Problem: Find area between $y=x$ and $y=x^2$ from $x=0$ to $x=1$.

Step 1: Compare curves on $0\leq x\leq 1$: x is above x^2.

Step 2: Set up area: $\int_0^1(x-x^2)dx$.

Step 3: Integrate: $x^2/2 - x^3/3$ from 0 to 1.

Step 4: Evaluate: $1/2 - 1/3 = 1/6$.

Memory key: Area is top minus bottom.

Kinematics

Problem: A cart starts from rest and accelerates at 2 m/s² for 5 s. Find displacement.

Step 1: Known: $u=0$, $a=2$ m/s², $t=5$ s, find s.

Step 2: Choose $s=ut+\frac{1}{2}at^2$ because u,a,t are known.

Step 3: Substitute: $s=0(5)+\frac{1}{2}(2)(5^2)$.

Step 4: Calculate: $s=25$ m.

Step 5: Final answer: 25 m.

Memory key: List variables before choosing equation.

Forces

Problem: A 10 kg box is pulled with 50 N on a frictionless surface. Find acceleration.

Step 1: Known: $m=10$ kg, net force=50 N.

Step 2: Use Newton's second law: $\Sigma F=ma$.

Step 3: Substitute: $50=10a$.

Step 4: Solve: $a=5$ m/s^2.

Memory key: Net force equals mass times acceleration.

Incline with friction

Problem: A block slides down an incline. Explain how to find acceleration.

Step 1: Draw weight mg downward, normal perpendicular to slope, friction up slope if sliding down.

Step 2: Resolve weight: $mg \sin\theta$ down slope and $mg \cos\theta$ perpendicular.

Step 3: Normal force $N=mg \cos\theta$ if no vertical/perpendicular acceleration.

Step 4: Kinetic friction $f_k=\mu_k N=\mu_k mg \cos\theta$.

Step 5: Net force down slope: $mg \sin\theta - \mu_k mg \cos\theta = ma$, so $a=g(\sin\theta-\mu_k\cos\theta)$.

Memory key: Resolve forces along and perpendicular to slope.

Energy

Problem: A 2 kg object falls from height 5 m. Find speed just before hitting ground, neglect air resistance.

Step 1: Initial energy: mgh. Final energy: $\frac{1}{2}mv^2$.

Step 2: Conservation: $mgh=\frac{1}{2}mv^2$.

Step 3: Mass cancels: $gh=\frac{1}{2}v^2$.

Step 4: Solve: $v=\sqrt{(2gh)}=\sqrt{(2\times9.8\times5)}\approx9.9$ m/s.

Memory key: Energy can avoid time calculations.

Momentum

Problem: A 1 kg cart moving 4 m/s sticks to a 3 kg cart at rest. Find final speed.

Step 1: System: both carts. Momentum conserved because they stick and external impulse is negligible.

Step 2: Initial momentum: $(1)(4)+(3)(0)=4$ kg m/s.

Step 3: Final mass: $1+3=4$ kg.

Step 4: Equation: $4=4v$, so $v=1$ m/s.

Memory key: For sticking collisions, final mass is combined.

Torque equilibrium

Problem: A force of 20 N acts 0.30 m from a pivot at 90°. Find torque.

Step 1: Use $\tau=rF\sin\theta$.

Step 2: Substitute $r=0.30$ m, $F=20$ N, $\theta=90°$, $\sin 90°=1$.

Step 3: $\tau=0.30\times20\times1=6$ N m.

Step 4: Direction depends on clockwise/counterclockwise rotation.

Memory key: Torque needs perpendicular lever arm.

Fluids

Problem: Find pressure increase at depth 2 m in water.

Step 1: Use $\Delta p=\rho gh$.

Step 2: For water, $\rho\approx1000$ kg/m^3; $g\approx9.8$ m/s^2; $h=2$ m.

Step 3: Substitute: $\Delta p=1000\times9.8\times2=19600$ Pa.

Step 4: Final: pressure increases by 1.96×10^4 Pa.

Memory key: Pressure increases with depth.

AP Calculus AB + AP Physics 1 Complete Practice Workbook

1,000 original legal practice questions with direct answers, detailed work, and memory keys. This edition converts caret notation into true superscript notation: x^2, x^3, t^2, r^2, v^2, $a^2 + b^2 = c^2$.

Notation rule: powers are written with superscripts, not caret symbols. Multiplication is written with × where needed, so it is easier to distinguish multiplication from exponentiation.

AP Calculus AB	500	Limits, derivatives, integrals, applications, and calculator-free reasoning
AP Physics 1	500	Kinematics, forces, energy, momentum, rotation, oscillations, fluids, and reasoning with units

AP Calculus AB

Unit 1: Limits and Continuity

Question 1

Problem: Evaluate the limit: lim as $x \to 3$ of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For $x \neq 3$, cancel $(x - 3)$, leaving $x + 3$. Substitute $x = 3$: $3 + 3 = 6$.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 2

Problem: Let $f(x) = 2x^3 + 3x^2$. Find $f'(2)$.

Direct Answer: $f'(2) = 36$

Detailed Work: Use the power rule: $d/dx[2x^3] = 6x^2$, and $d/dx[3x^2] = 6x^1$. Thus $f'(x) = 6x^2 + 6x^1$. Substitute $x=2$: $f'(2) = 36$.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 3

Problem: If $f(x)=(3x+2)^3$, find $f'(2)$.

Direct Answer: $f'(2) = 576$

Detailed Work: Use the chain rule. $f'(x) = 3(3x+2)^2 \times 3 = 9(3x+2)^2$. At $x=2$, the inside is 8, so $f'(2) = 9(8)^2 = 576$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 4

Problem: A particle has position $s(t) = 2t^3 - 3t^2 + 4t$. Find velocity and acceleration at $t=2$.

Direct Answer: $v(2) = 16$, $a(2) = 18$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 6t + 4$. Acceleration is $v'(t) = s''(t) = 12t - 6$. Substituting $t=2$ gives $v=16$ and $a=18$.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 5

Problem: Find the absolute minimum value of $f(x) = 2(x - 2)^2 + 3$.

Direct Answer: Minimum value = 3 at $x = 2$

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=2>0$, so it opens upward. The vertex is (2,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 6

Problem: Evaluate the definite integral from 0 to 2 of $(2x^2 + 2)\,dx$.

Direct Answer: 9.333

Detailed Work: Antiderivative: integral of $2x^2$ is $2/(3)\ x^3$, and integral of 2 is $2x$. Evaluate at 2 and subtract the value at 0: 9.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 7

Problem: Solve the differential equation $dy/dx = 2x$ with initial condition $y(0) = 3$.

Direct Answer: $y = (2/2)x^2 + 3$

Detailed Work: Integrate both sides with respect to x: $y = \text{integral } 2x\,dx = (2/2)x^2 + C$. Use $y(0)=3$: $3=0+C$, so $C=3$.

Memory Key: For $dy/dx = g(x)$, integrate $g(x)$ and use the initial condition.

Unit 8: Applications of Integration

Question 8

Problem: Find the area between $y=3$ and $y=x$ on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is $y=3$ and the bottom function is $y=x$ on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3 = 9 - 4.5 = 4.5$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 9

Problem: Find c so that f(x) is continuous at x = 3: $f(x)=x^2+c$ for $x<3$, and $f(x)=4x+3$ for $x\ge3$.

Direct Answer: c = 6

Detailed Work: For continuity, left value must equal right value at x=3. Left: $3^2 + c = 9 + c$. Right: 4(3) + 3 = 15. Set 9+c = 15, so c = 15 - 9 = 6.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 10

Problem: Let $f(x) = 3x^4 + 4x^3$. Find f'(3).

Direct Answer: f'(3) = 432

Detailed Work: Use the power rule: $d/dx[3x^4] = 12x^3$, and $d/dx[4x^3] = 12x^2$. Thus $f'(x) = 12x^3 + 12x^2$. Substitute x=3: f'(3) = 432.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 11

Problem: For the curve $x^2 + y^2 = 25$, find dy/dx at the point (3, 4).

Direct Answer: dy/dx = -3/4 = -0.75

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (3,4), dy/dx = -3/4.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 12

Problem: A particle has position $s(t) = 3t^3 - 4t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 62, a(3) = 46

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 8t + 5$. Acceleration is v'(t) = s''(t) = 18t - 8. Substituting t=3 gives v=62 and a=46.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 13

Problem: Find the absolute minimum value of $f(x) = 3(x - 3)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (3,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 14

Problem: Evaluate the definite integral from 0 to 3 of $(3x^3 + 3)$ dx.

Direct Answer: 69.75

Detailed Work: Antiderivative: integral of $3x^3$ is $3/(4)\ x^4$, and integral of 3 is 3x. Evaluate at 3 and subtract the value at 0: 69.75.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 15

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 16

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 17

Problem: Evaluate lim as x → 0 of sin(4x)/x.

Direct Answer: 4

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(4x)/x = 4 × [sin(4x)/(4x)]. The bracket approaches 1, so the limit is 4.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 18

Problem: Let $f(x) = 4x^5 + 5x^1$. Find f'(4).

Direct Answer: f'(4) = 5125

Detailed Work: Use the power rule: $d/dx[4x^5] = 20x^4$, and $d/dx[5x^1] = 5x^0$. Thus $f'(x) = 20x^4 + 5x^0$. Substitute x=4: f'(4) = 5125.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 19

Problem: Let $f(x)=e^x + 4x^2$. Find f'(4) exactly.

Direct Answer: $e^4 + 32$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(4)=e^4+32$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 20

Problem: A particle has position $s(t) = 4t^3 - 5t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 158, a(4) = 86

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 10t + 6$. Acceleration is v'(t) = s''(t) = 24t - 10. Substituting t=4 gives v=158 and a=86.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 21

Problem: Find the absolute minimum value of $f(x) = 4(x - 4)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (4,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 22

Problem: Evaluate the definite integral from 0 to 4 of $(4x^4 + 4)$ dx.

Direct Answer: 835.2

Detailed Work: Antiderivative: integral of $4x^4$ is $4/(5)\ x^5$, and integral of 4 is 4x. Evaluate at 4 and subtract the value at 0: 835.2.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 23

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 5.

Direct Answer: $y = (4/2)x^2 + 5$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2 + C$. Use y(0)=5: 5=0+C, so C=5.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 24

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 25

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel (x - 6), leaving x + 6. Substitute x = 6: 6 + 6 = 12.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 26

Problem: Let $f(x) = 5x^2 + 6x^2$. Find f'(1).

Direct Answer: f'(1) = 22

Detailed Work: Use the power rule: $d/dx[5x^2] = 10x^1$, and $d/dx[6x^2] = 12x^1$. Thus $f'(x) = 10x^1 + 12x^1$. Substitute x=1: f'(1) = 22.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 27

Problem: If $f(x)=(6x+1)^3$, find f'(2).

Direct Answer: f'(2) = 3042

Detailed Work: Use the chain rule. $f'(x) = 3(6x+1)^2 \times 6 = 18(6x+1)^2$. At x=2, the inside is 13, so $f'(2) = 18(13)^2 = 3042$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 28

Problem: A particle has position $s(t) = 1t^3 - 6t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 22, a(5) = 18

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 12t + 7$. Acceleration is v'(t) = s''(t) = 6t - 12. Substituting t=5 gives v=22 and a=18.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 29

Problem: Find the absolute minimum value of $f(x) = 1(x - 5)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (5,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 30

Problem: Evaluate the definite integral from 0 to 1 of ($5x^1 + 5$) dx.

Direct Answer: 7.5

Detailed Work: Antiderivative: integral of $5x^1$ is 5/(2) x^2, and integral of 5 is 5x. Evaluate at 1 and subtract the value at 0: 7.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 31

Problem: A quantity satisfies dy/dt = 1y and y(0) = 6. Write y(t).

Direct Answer: $y(t) = 6e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=6, C=6. Thus $y(t)=6e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 32

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = [2x - x^2/2]_0^2 = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 33

Problem: Find c so that f(x) is continuous at x = 6: $f(x)=x^2+c$ for x<6, and f(x)=7x+3 for x≥6.

Direct Answer: c = 9

Detailed Work: For continuity, left value must equal right value at x=6. Left: 6^2 + c = 36 + c. Right: 7(6) + 3 = 45. Set 36+c = 45, so c = 45 - 36 = 9.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 34

Problem: Let $f(x) = 6x^3 + 2x^3$. Find f'(2).

Direct Answer: f'(2) = 96

Detailed Work: Use the power rule: $d/dx[6x^3] = 18x^2$, and $d/dx[2x^3] = 6x^2$. Thus $f'(x) = 18x^2 + 6x^2$. Substitute x=2: f'(2) = 96.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 35

Problem: For the curve $x^2 + y^2 = 5$, find dy/dx at the point (1, 2).

Direct Answer: dy/dx = -½ = -0.5

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (1,2), dy/dx = -½.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 36

Problem: A particle has position $s(t) = 2t^3 - 7t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = -5, a(1) = -2

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 14t + 3$. Acceleration is v'(t) = s''(t) = 12t - 14. Substituting t=1 gives v=-5 and a=-2.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 37

Problem: Find the absolute minimum value of $f(x) = 2(x - 6)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (6,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 38

Problem: Evaluate the definite integral from 0 to 2 of $(1x^2 + 6)$ dx.

Direct Answer: 14.667

Detailed Work: Antiderivative: integral of $1x^2$ is $1/(3)\ x^3$, and integral of 6 is 6x. Evaluate at 2 and subtract the value at 0: 14.667.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 39

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 7.

Direct Answer: $y = (6/2)x^2 + 7$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2$ + C. Use y(0)=7: 7=0+C, so C=7.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 40

Problem: Find the area between y=3 and y=x on the interval $0 \leq x \leq 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 41

Problem: Evaluate lim as $x \to 0$ of sin(7x)/x.

Direct Answer: 7

Detailed Work: Use the standard limit lim as $u \to 0$ of sin(u)/u = 1. Write sin(7x)/x = 7 × [sin(7x)/(7x)]. The bracket approaches 1, so the limit is 7.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 42

Problem: Let $f(x) = 1x^4 + 3x^1$. Find f'(3).

Direct Answer: f'(3) = 111

Detailed Work: Use the power rule: $d/dx[1x^4] = 4x^3$, and $d/dx[3x^1] = 3x^0$. Thus $f'(x) = 4x^3 + 3x^0$. Substitute x=3: f'(3) = 111.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 43

Problem: Let $f(x)=e^x + 1x^2$. Find f'(3) exactly.

Direct Answer: $e^3 + 6$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore $f'(x)=e^x+2x$, so $f'(3)=e^3+6$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 44

Problem: A particle has position $s(t) = 3t^3 - 2t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 32, a(2) = 32

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 4t + 4$. Acceleration is $v'(t) = s''(t) = 18t - 4$. Substituting t=2 gives v=32 and a=32.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 45

Problem: Find the absolute minimum value of $f(x) = 3(x - 1)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (1,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 46

Problem: Evaluate the definite integral from 0 to 3 of $(2x^3 + 1)$ dx.

Direct Answer: 43.5

Detailed Work: Antiderivative: integral of $2x^3$ is $2/(4)\ x^4$, and integral of 1 is 1x. Evaluate at 3 and subtract the value at 0: 43.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 47

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: $y(t) = 8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus $y(t)=8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 48

Problem: Find the area between y=4 and y=x on the interval $0 \le x \le 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 49

Problem: Evaluate the limit: lim as x → 9 of $(x^2 - 81)/(x - 9)$.

Direct Answer: 18

Detailed Work: Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$. For $x \neq 9$, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 50

Problem: Let $f(x) = 2x^5 + 4x^2$. Find f'(4).

Direct Answer: f'(4) = 2592

Detailed Work: Use the power rule: $d/dx[2x^5] = 10x^4$, and $d/dx[4x^2] = 8x^1$. Thus $f'(x) = 10x^4 + 8x^1$. Substitute x=4: f'(4) = 2592.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 51

Problem: If $f(x)=(4x+4)^3$, find f'(2).

Direct Answer: f'(2) = 1728

Detailed Work: Use the chain rule. $f'(x) = 3(4x+4)^2 \times 4 = 12(4x+4)^2$. At x=2, the inside is 12, so $f'(2) = 12(12)^2 = 1728$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 52

Problem: A particle has position $s(t) = 4t^3 - 3t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 95, a(3) = 66

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 6t + 5$. Acceleration is $v'(t) = s''(t) = 24t - 6$. Substituting t=3 gives v=95 and a=66.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 53

Problem: Find the absolute minimum value of $f(x) = 4(x - 2)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (2,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 54

Problem: Evaluate the definite integral from 0 to 4 of $(3x^4 + 2)$ dx.

Direct Answer: 622.4

Detailed Work: Antiderivative: integral of $3x^4$ is $3/(5)\ x^5$, and integral of 2 is 2x. Evaluate at 4 and subtract the value at 0: 622.4.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 55

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 9.

Direct Answer: $y = (2/2)x^2 + 9$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use y(0)=9: 9=0+C, so C=9.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 56

Problem: Find the area between y=5 and y=x on the interval 0 ≤ x ≤ 5.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 57

Problem: Find c so that f(x) is continuous at x = 1: $f(x)=x^2+c$ for x<1, and f(x)=3x+3 for x≥1.

Direct Answer: c = 5

Detailed Work: For continuity, left value must equal right value at x=1. Left: $1^2 + c = 1 + c$. Right: 3(1) + 3 = 6. Set 1+c = 6, so c = 6 - 1 = 5.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 58

Problem: Let $f(x) = 3x^2 + 5x^3$. Find f'(1).

Direct Answer: f'(1) = 21

Detailed Work: Use the power rule: $d/dx[3x^2] = 6x^1$, and $d/dx[5x^3] = 15x^2$. Thus $f'(x) = 6x^1 + 15x^2$. Substitute x=1: f'(1) = 21.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 59

Problem: For the curve $x^2 + y^2 = 41$, find dy/dx at the point (4, 5).

Direct Answer: dy/dx = -4/5 = -0.8

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (4,5), dy/dx = -4/5.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 60

Problem: A particle has position $s(t) = 1t^3 - 4t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 22, a(4) = 16

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 8t + 6$. Acceleration is v'(t) = s''(t) = 6t - 8. Substituting t=4 gives v=22 and a=16.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 61

Problem: Find the absolute minimum value of $f(x) = 1(x - 3)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (3,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 62

Problem: Evaluate the definite integral from 0 to 1 of $(4x^1 + 3)$ dx.

Direct Answer: 5

Detailed Work: Antiderivative: integral of $4x^1$ is $4/(2) x^2$, and integral of 3 is 3x. Evaluate at 1 and subtract the value at 0: 5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 63

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 64

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2$ = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 65

Problem: Evaluate lim as x → 0 of sin(10x)/x.

Direct Answer: 10

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(10x)/x = 10 × [sin(10x)/(10x)]. The bracket approaches 1, so the limit is 10.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 66

Problem: Let $f(x) = 4x^3 + 6x^1$. Find f'(2).

Direct Answer: f'(2) = 54

Detailed Work: Use the power rule: $d/dx[4x^3] = 12x^2$, and $d/dx[6x^1] = 6x^0$. Thus $f'(x) = 12x^2 + 6x^0$. Substitute x=2: f'(2) = 54.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 67

Problem: Let $f(x)=e^x + 4x^2$. Find f'(2) exactly.

Direct Answer: $e^2 + 16$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(2)=e^2+16$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 68

Problem: A particle has position $s(t) = 2t^3 - 5t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 107, a(5) = 50

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 10t + 7$. Acceleration is v'(t) = s''(t) = 12t - 10. Substituting t=5 gives v=107 and a=50.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 69

Problem: Find the absolute minimum value of $f(x) = 2(x - 4)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (4,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 70

Problem: Evaluate the definite integral from 0 to 2 of $(5x^2 + 4)$ dx.

Direct Answer: 21.333

Detailed Work: Antiderivative: integral of $5x^2$ is $5/(3) x^3$, and integral of 4 is 4x. Evaluate at 2 and subtract the value at 0: 21.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 71

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 2.

Direct Answer: $y = (4/2)x^2 + 2$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2$ + C. Use y(0)=2: 2=0+C, so C=2.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 72

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 73

Problem: Evaluate the limit: lim as x → 3 of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For x ≠ 3, cancel (x - 3), leaving x + 3. Substitute x = 3: 3 + 3 = 6.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 74

Problem: Let $f(x) = 5x^4 + 2x^2$. Find f'(3).

Direct Answer: f'(3) = 552

Detailed Work: Use the power rule: $d/dx[5x^4] = 20x^3$, and $d/dx[2x^2] = 4x^1$. Thus $f'(x) = 20x^3 + 4x^1$. Substitute x=3: f'(3) = 552.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 75

Problem: If $f(x)=(2x+3)^3$, find f'(2).

Direct Answer: f'(2) = 294

Detailed Work: Use the chain rule. $f'(x) = 3(2x+3)^2 \times 2 = 6(2x+3)^2$. At x=2, the inside is 7, so $f'(2) = 6(7)^2 = 294$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 76

Problem: A particle has position $s(t) = 3t^3 - 6t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 0, a(1) = 6

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 12t + 3$. Acceleration is v'(t) = s''(t) = 18t - 12. Substituting t=1 gives v=0 and a=6.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 77

Problem: Find the absolute minimum value of $f(x) = 3(x - 5)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (5,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 78

Problem: Evaluate the definite integral from 0 to 3 of $(1x^3 + 5)$ dx.

Direct Answer: 35.25

Detailed Work: Antiderivative: integral of $1x^3$ is $1/(4)\ x^4$, and integral of 5 is 5x. Evaluate at 3 and subtract the value at 0: 35.25.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 79

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 80

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = [4x - x^2/2]_0^4 = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 81

Problem: Find c so that f(x) is continuous at x = 4: f(x)=x^2+c for x<4, and f(x)=6x+3 for x≥4.

Direct Answer: c = 11

Detailed Work: For continuity, left value must equal right value at x=4. Left: 4^2 + c = 16 + c. Right: 6(4) + 3 = 27. Set 16+c = 27, so c = 27 - 16 = 11.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 82

Problem: Let f(x) = $6x^5 + 3x^3$. Find f'(4).

Direct Answer: f'(4) = 7824

Detailed Work: Use the power rule: d/dx[$6x^5$] = $30x^4$, and d/dx[$3x^3$] = $9x^2$. Thus f'(x) = $30x^4 + 9x^2$. Substitute x=4: f'(4) = 7824.

Memory Key: Power rule: d/dx(x^n) = n x^{n-1}.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 83

Problem: For the curve $x^2 + y^2 = 13$, find dy/dx at the point (2, 3).

Direct Answer: dy/dx = -2/3 = -0.667

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (2,3), dy/dx = -2/3.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 84

Problem: A particle has position s(t) = $4t^3 - 7t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 24, a(2) = 34

Detailed Work: Velocity is the derivative of position: v(t) = s'(t) = $12t^2 - 14t + 4$. Acceleration is v'(t) = s''(t) = 24t - 14. Substituting t=2 gives v=24 and a=34.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 85

Problem: Find the absolute minimum value of f(x) = $4(x - 6)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (6,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 86

Problem: Evaluate the definite integral from 0 to 4 of ($2x^4 + 6$) dx.

Direct Answer: 433.6

Detailed Work: Antiderivative: integral of $2x^4$ is 2/(5) x^5, and integral of 6 is 6x. Evaluate at 4 and subtract the value at 0: 433.6.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 87

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 4.

Direct Answer: y = (6/2)x^2 + 4

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = (6/2)x^2 + C. Use y(0)=4: 4=0+C, so C=4.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 88

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 89

Problem: Evaluate lim as $x \to 0$ of sin(3x)/x.

Direct Answer: 3

Detailed Work: Use the standard limit lim as $u \to 0$ of sin(u)/u = 1. Write sin(3x)/x = 3 × [sin(3x)/(3x)]. The bracket approaches 1, so the limit is 3.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 90

Problem: Let $f(x) = 1x^2 + 4x^1$. Find f'(1).

Direct Answer: f'(1) = 6

Detailed Work: Use the power rule: $d/dx[1x^2] = 2x^1$, and $d/dx[4x^1] = 4x^0$. Thus $f'(x) = 2x^1 + 4x^0$. Substitute x=1: f'(1) = 6.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 91

Problem: Let $f(x)=e^x + 1x^2$. Find f'(1) exactly.

Direct Answer: $e^1 + 2$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore $f'(x)=e^x+2x$, so $f'(1)=e^1+2$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 92

Problem: A particle has position $s(t) = 1t^3 - 2t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 20, a(3) = 14

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 4t + 5$. Acceleration is v'(t) = s''(t) = 6t - 4. Substituting t=3 gives v=20 and a=14.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 93

Problem: Find the absolute minimum value of $f(x) = 1(x - 1)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (1,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 94

Problem: Evaluate the definite integral from 0 to 1 of $(3x^1 + 1)$ dx.

Direct Answer: 2.5

Detailed Work: Antiderivative: integral of $3x^1$ is $3/(2)\ x^2$, and integral of 1 is 1x. Evaluate at 1 and subtract the value at 0: 2.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 95

Problem: A quantity satisfies $dy/dt = 1y$ and $y(0) = 6$. Write $y(t)$.

Direct Answer: $y(t) = 6e^{1}t$

Detailed Work: The equation $dy/dt = ky$ has exponential solution y = Cekt. Since $y(0)=6$, $C=6$. Thus $y(t)=6e^{1}t$.

Memory Key: $dy/dt = ky$ gives exponential growth/decay.

Unit 8: Applications of Integration

Question 96

Problem: Find the area between $y=2$ and $y=x$ on the interval $0 \leq x \leq 2$.

Direct Answer: 2

Detailed Work: The top function is $y=2$ and the bottom function is $y=x$ on [0,2]. Area = integral from 0 to 2 of (2-x) dx = [2x - $x^2/2$]_0^2 = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 97

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel $(x - 6)$, leaving $x + 6$. Substitute $x = 6$: $6 + 6 = 12$.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 98

Problem: Let $f(x) = 2x^3 + 5x^2$. Find $f'(2)$.

Direct Answer: $f'(2) = 44$

Detailed Work: Use the power rule: $d/dx[2x^3] = 6x^2$, and $d/dx[5x^2] = 10x^1$. Thus $f'(x) = 6x^2 + 10x^1$. Substitute $x=2$: $f'(2) = 44$.

Memory Key: Power rule: $d/dx(x^n) = n\, x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 99

Problem: If $f(x)=(5x+2)^3$, find $f'(2)$.

Direct Answer: $f'(2) = 2160$

Detailed Work: Use the chain rule. $f'(x) = 3(5x+2)^2 \times 5 = 15(5x+2)^2$. At $x=2$, the inside is 12, so $f'(2) = 15(12)^2 = 2160$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 100

Problem: A particle has position $s(t) = 2t^3 - 3t^2 + 6t$. Find velocity and acceleration at $t=4$.

Direct Answer: $v(4) = 78$, $a(4) = 42$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 6t + 6$. Acceleration is $v'(t) = s''(t) = 12t - 6$. Substituting $t=4$ gives $v=78$ and $a=42$.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 101

Problem: Find the absolute minimum value of $f(x) = 2(x - 2)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=2>0$, so it opens upward. The vertex is (2,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 102

Problem: Evaluate the definite integral from 0 to 2 of $(4x^2 + 2)$ dx.

Direct Answer: 14.667

Detailed Work: Antiderivative: integral of $4x^2$ is $4/(3) x^3$, and integral of 2 is 2x. Evaluate at 2 and subtract the value at 0: 14.667.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 103

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 6.

Direct Answer: $y = (2/2)x^2 + 6$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2$ + C. Use y(0)=6: 6=0+C, so C=6.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 104

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3 = 9 - 4.5 = 4.5$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 105

Problem: Find c so that f(x) is continuous at x = 7: $f(x)=x^2+c$ for $x<7$, and f(x)=2x+3 for $x \ge 7$.

Direct Answer: c = -32

Detailed Work: For continuity, left value must equal right value at x=7. Left: $7^2 + c = 49 + c$. Right: 2(7) + 3 = 17. Set 49+c = 17, so c = 17 - 49 = -32.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 106

Problem: Let $f(x) = 3x^4 + 6x^3$. Find f'(3).

Direct Answer: f'(3) = 486

Detailed Work: Use the power rule: $d/dx[3x^4] = 12x^3$, and $d/dx[6x^3] = 18x^2$. Thus $f'(x) = 12x^3 + 18x^2$. Substitute x=3: f'(3) = 486.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 107

Problem: For the curve $x^2 + y^2 = 61$, find dy/dx at the point (5, 6).

Direct Answer: dy/dx = -5/6 = -0.833

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (5,6), dy/dx = -5/6.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 108

Problem: A particle has position $s(t) = 3t^3 - 4t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 192, a(5) = 82

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 8t + 7$. Acceleration is v'(t) = s''(t) = 18t - 8. Substituting t=5 gives v=192 and a=82.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 109

Problem: Find the absolute minimum value of $f(x) = 3(x - 3)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (3,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 110

Problem: Evaluate the definite integral from 0 to 3 of ($5x^3$ + 3) dx.

Direct Answer: 110.25

Detailed Work: Antiderivative: integral of $5x^3$ is 5/(4) x^4, and integral of 3 is 3x. Evaluate at 3 and subtract the value at 0: 110.25.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 111

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: y(t) = $8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus y(t)=$8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 112

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = [4x - x^2/2]_0^4 = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 113

Problem: Evaluate lim as x → 0 of sin(6x)/x.

Direct Answer: 6

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(6x)/x = 6 × [sin(6x)/(6x)]. The bracket approaches 1, so the limit is 6.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 114

Problem: Let f(x) = $4x^5$ + $2x^1$. Find f'(4).

Direct Answer: f'(4) = 5122

Detailed Work: Use the power rule: d/dx[$4x^5$] = $20x^4$, and d/dx[$2x^1$] = $2x^0$. Thus f'(x) = $20x^4$ + $2x^0$. Substitute x=4: f'(4) = 5122.

Memory Key: Power rule: d/dx(x^n) = n x^{n-1}.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 115

Problem: Let f(x)=e^x + $4x^2$. Find f'(4) exactly.

Direct Answer: e^4 + 32

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore f'(x)=e^x+8x, so f'(4)=e^4+32.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 116

Problem: A particle has position s(t) = $4t^3$ - $5t^2$ + 3t. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 5, a(1) = 14

Detailed Work: Velocity is the derivative of position: v(t) = s'(t) = $12t^2$ - 10t + 3. Acceleration is v'(t) = s''(t) = 24t - 10. Substituting t=1 gives v=5 and a=14.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 117

Problem: Find the absolute minimum value of $f(x) = 4(x - 4)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (4,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 118

Problem: Evaluate the definite integral from 0 to 4 of $(1x^4 + 4)$ dx.

Direct Answer: 220.8

Detailed Work: Antiderivative: integral of $1x^4$ is $1/(5)\ x^5$, and integral of 4 is 4x. Evaluate at 4 and subtract the value at 0: 220.8.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 119

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 8.

Direct Answer: $y = (4/2)x^2 + 8$

Detailed Work: Integrate both sides with respect to x: $y = \text{integral } 4x\, dx = (4/2)x^2 + C$. Use y(0)=8: 8=0+C, so C=8.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 120

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 121

Problem: Evaluate the limit: lim as x → 9 of $(x^2 - 81)/(x - 9)$.

Direct Answer: 18

Detailed Work: Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$. For $x \neq 9$, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 122

Problem: Let $f(x) = 5x^2 + 3x^2$. Find f'(1).

Direct Answer: f'(1) = 16

Detailed Work: Use the power rule: $d/dx[5x^2] = 10x^1$, and $d/dx[3x^2] = 6x^1$. Thus $f'(x) = 10x^1 + 6x^1$. Substitute x=1: f'(1) = 16.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 123

Problem: If $f(x)=(3x+1)^3$, find f'(2).

Direct Answer: f'(2) = 441

Detailed Work: Use the chain rule. $f'(x) = 3(3x+1)^2 \times 3 = 9(3x+1)^2$. At x=2, the inside is 7, so $f'(2) = 9(7)^2 = 441$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 124

Problem: A particle has position $s(t) = 1t^3 - 6t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = -8, a(2) = 0

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 12t + 4$. Acceleration is $v'(t) = s''(t) = 6t - 12$. Substituting t=2 gives v=-8 and a=0.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 125

Problem: Find the absolute minimum value of $f(x) = 1(x - 5)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (5,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 126

Problem: Evaluate the definite integral from 0 to 1 of $(2x^1 + 5)$ dx.

Direct Answer: 6

Detailed Work: Antiderivative: integral of $2x^1$ is $2/(2)\ x^2$, and integral of 5 is 5x. Evaluate at 1 and subtract the value at 0: 6.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 127

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 128

Problem: Find the area between y=2 and y=x on the interval $0 \le x \le 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = [2x - $x^2/2$]_0^2 = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 129

Problem: Find c so that f(x) is continuous at x = 2: $f(x)=x^2+c$ for x<2, and f(x)=5x+3 for x≥2.

Direct Answer: c = 9

Detailed Work: For continuity, left value must equal right value at x=2. Left: $2^2 + c = 4 + c$. Right: 5(2) + 3 = 13. Set 4+c = 13, so c = 13 - 4 = 9.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 130

Problem: Let $f(x) = 6x^3 + 4x^3$. Find f'(2).

Direct Answer: f'(2) = 120

Detailed Work: Use the power rule: $d/dx[6x^3] = 18x^2$, and $d/dx[4x^3] = 12x^2$. Thus $f'(x) = 18x^2 + 12x^2$. Substitute x=2: f'(2) = 120.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 131

Problem: For the curve $x^2 + y^2 = 25$, find dy/dx at the point (3, 4).

Direct Answer: dy/dx = -3/4 = -0.75

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (3,4), dy/dx = -3/4.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 132

Problem: A particle has position $s(t) = 2t^3 - 7t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 17, a(3) = 22

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 14t + 5$. Acceleration is v'(t) = s''(t) = 12t - 14. Substituting t=3 gives v=17 and a=22.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 133

Problem: Find the absolute minimum value of $f(x) = 2(x - 6)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (6,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 134

Problem: Evaluate the definite integral from 0 to 2 of $(3x^2 + 6)$ dx.

Direct Answer: 20

Detailed Work: Antiderivative: integral of $3x^2$ is 3/(3) x^3, and integral of 6 is 6x. Evaluate at 2 and subtract the value at 0: 20.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 135

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 10.

Direct Answer: $y = (6/2)x^2 + 10$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2$ + C. Use y(0)=10: 10=0+C, so C=10.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 136

Problem: Find the area between y=3 and y=x on the interval 0 ≤ x ≤ 3.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = [3x - x^2/2]_0^3 = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 137

Problem: Evaluate lim as x → 0 of sin(9x)/x.

Direct Answer: 9

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(9x)/x = 9 × [sin(9x)/(9x)]. The bracket approaches 1, so the limit is 9.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 138

Problem: Let $f(x) = 1x^4 + 5x^1$. Find f'(3).

Direct Answer: $f'(3) = 113$

Detailed Work: Use the power rule: $d/dx[1x^4] = 4x^3$, and $d/dx[5x^1] = 5x^0$. Thus $f'(x) = 4x^3 + 5x^0$. Substitute x=3: $f'(3) = 113$.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 139

Problem: Let $f(x)=e^x + 1x^2$. Find $f'(3)$ exactly.

Direct Answer: $e^3 + 6$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is $2x$. Therefore $f'(x)=e^x+2x$, so $f'(3)=e^3+6$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes $2x$.

Unit 4: Contextual Applications of Differentiation

Question 140

Problem: A particle has position $s(t) = 3t^3 - 2t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: $v(4) = 134$, $a(4) = 68$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 4t + 6$. Acceleration is $v'(t) = s''(t) = 18t - 4$. Substituting t=4 gives v=134 and a=68.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 141

Problem: Find the absolute minimum value of $f(x) = 3(x - 1)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=3>0$, so it opens upward. The vertex is (1,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 142

Problem: Evaluate the definite integral from 0 to 3 of $(4x^3 + 1)$ dx.

Direct Answer: 84

Detailed Work: Antiderivative: integral of $4x^3$ is $4/(4)\ x^4$, and integral of 1 is 1x. Evaluate at 3 and subtract the value at 0: 84.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 143

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 144

Problem: Find the area between y=4 and y=x on the interval $0 \le x \le 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4 = 16 - 8 = 8$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 145

Problem: Evaluate the limit: lim as x → 3 of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For $x \neq 3$, cancel (x - 3), leaving x + 3. Substitute x = 3: 3 + 3 = 6.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 146

Problem: Let $f(x) = 2x^5 + 6x^2$. Find f'(4).

Direct Answer: f'(4) = 2608

Detailed Work: Use the power rule: $d/dx[2x^5] = 10x^4$, and $d/dx[6x^2] = 12x^1$. Thus $f'(x) = 10x^4 + 12x^1$. Substitute x=4: f'(4) = 2608.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 147

Problem: If $f(x)=(6x+4)^3$, find f'(2).

Direct Answer: f'(2) = 4608

Detailed Work: Use the chain rule. $f'(x) = 3(6x+4)^2 \times 6 = 18(6x+4)^2$. At x=2, the inside is 16, so $f'(2) = 18(16)^2 = 4608$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 148

Problem: A particle has position $s(t) = 4t^3 - 3t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 277, a(5) = 114

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 6t + 7$. Acceleration is v'(t) = s''(t) = 24t - 6. Substituting t=5 gives v=277 and a=114.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 149

Problem: Find the absolute minimum value of $f(x) = 4(x - 2)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (2,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 150

Problem: Evaluate the definite integral from 0 to 4 of $(5x^4 + 2)$ dx.

Direct Answer: 1032

Detailed Work: Antiderivative: integral of $5x^4$ is $5/(5) x^5$, and integral of 2 is 2x. Evaluate at 4 and subtract the value at 0: 1032.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 151

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 3.

Direct Answer: $y = (2/2)x^2 + 3$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use y(0)=3: 3=0+C, so C=3.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 152

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 153

Problem: Find c so that f(x) is continuous at x = 5: $f(x)=x^2+c$ for $x<5$, and $f(x)=8x+3$ for $x\geq5$.

Direct Answer: c = 18

Detailed Work: For continuity, left value must equal right value at x=5. Left: $5^2 + c = 25 + c$. Right: 8(5) + 3 = 43. Set 25+c = 43, so c = 43 - 25 = 18.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 154

Problem: Let $f(x) = 3x^2 + 2x^3$. Find f'(1).

Direct Answer: f'(1) = 12

Detailed Work: Use the power rule: $d/dx[3x^2] = 6x^1$, and $d/dx[2x^3] = 6x^2$. Thus $f'(x) = 6x^1 + 6x^2$. Substitute x=1: f'(1) = 12.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 155

Problem: For the curve $x^2 + y^2 = 5$, find dy/dx at the point (1, 2).

Direct Answer: dy/dx = -½ = -0.5

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (1,2), dy/dx = -½.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 156

Problem: A particle has position $s(t) = 1t^3 - 4t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = -2, a(1) = -2

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 8t + 3$. Acceleration is v'(t) = s''(t) = 6t - 8. Substituting t=1 gives v=-2 and a=-2.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 157

Problem: Find the absolute minimum value of $f(x) = 1(x - 3)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (3,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 158

Problem: Evaluate the definite integral from 0 to 1 of $(1x^1 + 3)$ dx.

Direct Answer: 3.5

Detailed Work: Antiderivative: integral of $1x^1$ is $1/(2)\ x^2$, and integral of 3 is 3x. Evaluate at 1 and subtract the value at 0: 3.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 159

Problem: A quantity satisfies dy/dt = 1y and y(0) = 6. Write y(t).

Direct Answer: $y(t) = 6e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=6, C=6. Thus $y(t)=6e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 160

Problem: Find the area between y=2 and y=x on the interval $0 \leq x \leq 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 161

Problem: Evaluate lim as $x \to 0$ of sin(2x)/x.

Direct Answer: 2

Detailed Work: Use the standard limit lim as $u \to 0$ of sin(u)/u = 1. Write sin(2x)/x = 2 × [sin(2x)/(2x)]. The bracket approaches 1, so the limit is 2.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 162

Problem: Let $f(x) = 4x^3 + 3x^1$. Find f'(2).

Direct Answer: f'(2) = 51

Detailed Work: Use the power rule: $d/dx[4x^3] = 12x^2$, and $d/dx[3x^1] = 3x^0$. Thus $f'(x) = 12x^2 + 3x^0$. Substitute x=2: f'(2) = 51.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 163

Problem: Let $f(x)=e^x + 4x^2$. Find f'(2) exactly.

Direct Answer: $e^2 + 16$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(2)=e^2+16$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 164

Problem: A particle has position $s(t) = 2t^3 - 5t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 8, a(2) = 14

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 10t + 4$. Acceleration is v'(t) = s''(t) = 12t - 10. Substituting t=2 gives v=8 and a=14.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 165

Problem: Find the absolute minimum value of $f(x) = 2(x - 4)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (4,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 166

Problem: Evaluate the definite integral from 0 to 2 of $(2x^2 + 4)$ dx.

Direct Answer: 13.333

Detailed Work: Antiderivative: integral of $2x^2$ is $2/(3) x^3$, and integral of 4 is 4x. Evaluate at 2 and subtract the value at 0: 13.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 167

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 5.

Direct Answer: $y = (4/2)x^2 + 5$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2 + C$. Use y(0)=5: 5=0+C, so C=5.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 168

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3 = 9 - 4.5 = 4.5$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 169

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel (x - 6), leaving x + 6. Substitute x = 6: 6 + 6 = 12.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 170

Problem: Let $f(x) = 5x^4 + 4x^2$. Find f'(3).

Direct Answer: f'(3) = 564

Detailed Work: Use the power rule: $d/dx[5x^4] = 20x^3$, and $d/dx[4x^2] = 8x^1$. Thus $f'(x) = 20x^3 + 8x^1$. Substitute x=3: f'(3) = 564.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 171

Problem: If $f(x)=(4x+3)^3$, find f'(2).

Direct Answer: f'(2) = 1452

Detailed Work: Use the chain rule. $f'(x) = 3(4x+3)^2 \times 4 = 12(4x+3)^2$. At x=2, the inside is 11, so $f'(2) = 12(11)^2 = 1452$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 172

Problem: A particle has position $s(t) = 3t^3 - 6t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 50, a(3) = 42

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 12t + 5$. Acceleration is v'(t) = s''(t) = 18t - 12. Substituting t=3 gives v=50 and a=42.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 173

Problem: Find the absolute minimum value of $f(x) = 3(x - 5)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (5,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 174

Problem: Evaluate the definite integral from 0 to 3 of $(3x^3 + 5)$ dx.

Direct Answer: 75.75

Detailed Work: Antiderivative: integral of $3x^3$ is $3/(4)\ x^4$, and integral of 5 is 5x. Evaluate at 3 and subtract the value at 0: 75.75.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 175

Problem: A quantity satisfies $dy/dt = 3y$ and $y(0) = 8$. Write $y(t)$.

Direct Answer: $y(t) = 8e^{3t}$

Detailed Work: The equation $dy/dt = ky$ has exponential solution $y = Cekt$. Since $y(0)=8$, $C=8$. Thus $y(t)=8e^{3t}$.

Memory Key: $dy/dt = ky$ gives exponential growth/decay.

Unit 8: Applications of Integration

Question 176

Problem: Find the area between $y=4$ and $y=x$ on the interval $0 \leq x \leq 4$.

Direct Answer: 8

Detailed Work: The top function is $y=4$ and the bottom function is $y=x$ on $[0,4]$. Area = integral from 0 to 4 of $(4-x)\,dx = [4x - x^2/2]\_0^4 = 16 - 8 = 8$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 177

Problem: Find c so that f(x) is continuous at $x = 8$: $f(x)=x^2+c$ for $x<8$, and $f(x)=4x+3$ for $x\geq 8$.

Direct Answer: $c = -29$

Detailed Work: For continuity, left value must equal right value at $x=8$. Left: $8^2 + c = 64 + c$. Right: $4(8) + 3 = 35$. Set $64+c = 35$, so $c = 35 - 64 = -29$.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 178

Problem: Let $f(x) = 6x^5 + 5x^3$. Find $f'(4)$.

Direct Answer: $f'(4) = 7920$

Detailed Work: Use the power rule: $d/dx[6x^5] = 30x^4$, and $d/dx[5x^3] = 15x^2$. Thus $f'(x) = 30x^4 + 15x^2$. Substitute $x=4$: $f'(4) = 7920$.

Memory Key: Power rule: $d/dx(x^n) = n\,x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 179

Problem: For the curve $x^2 + y^2 = 41$, find dy/dx at the point $(4, 5)$.

Direct Answer: $dy/dx = -4/5 = -0.8$

Detailed Work: Differentiate implicitly: $2x + 2y(dy/dx) = 0$. Solve for dy/dx: $dy/dx = -x/y$. At $(4,5)$, $dy/dx = -4/5$.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 180

Problem: A particle has position $s(t) = 4t^3 - 7t^2 + 6t$. Find velocity and acceleration at $t=4$.

Direct Answer: $v(4) = 142$, $a(4) = 82$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 14t + 6$. Acceleration is $v'(t) = s''(t) = 24t - 14$. Substituting $t=4$ gives $v=142$ and $a=82$.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 181

Problem: Find the absolute minimum value of $f(x) = 4(x - 6)^2 + 4$.

Direct Answer: Minimum value = 4 at $x = 6$

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=4>0$, so it opens upward. The vertex is $(6,4)$, and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 182

Problem: Evaluate the definite integral from 0 to 4 of ($4x^4 + 6$) dx.

Direct Answer: 843.2

Detailed Work: Antiderivative: integral of $4x^4$ is 4/(5) x^5, and integral of 6 is 6x. Evaluate at 4 and subtract the value at 0: 843.2.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 183

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 7.

Direct Answer: y = (6/2)x^2 + 7

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = (6/2)x^2 + C. Use y(0)=7: 7=0+C, so C=7.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 184

Problem: Find the area between y=5 and y=x on the interval 0 ≤ x ≤ 5.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = [5x - x^2/2]_0^5 = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 185

Problem: Evaluate lim as x → 0 of sin(5x)/x.

Direct Answer: 5

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(5x)/x = 5 × [sin(5x)/(5x)]. The bracket approaches 1, so the limit is 5.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 186

Problem: Let f(x) = $1x^2 + 6x^1$. Find f'(1).

Direct Answer: f'(1) = 8

Detailed Work: Use the power rule: d/dx[$1x^2$] = $2x^1$, and d/dx[$6x^1$] = $6x^0$. Thus f'(x) = $2x^1 + 6x^0$. Substitute x=1: f'(1) = 8.

Memory Key: Power rule: d/dx(x^n) = n x^{n-1}.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 187

Problem: Let f(x)=$e^x + 1x^2$. Find f'(1) exactly.

Direct Answer: e^1 + 2

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore f'(x)=e^x+2x, so f'(1)=e^1+2.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 188

Problem: A particle has position s(t) = $1t^3 - 2t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 62, a(5) = 26

Detailed Work: Velocity is the derivative of position: v(t) = s'(t) = $3t^2 - 4t + 7$. Acceleration is v'(t) = s''(t) = 6t - 4. Substituting t=5 gives v=62 and a=26.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 189

Problem: Find the absolute minimum value of $f(x) = 1(x - 1)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (1,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 190

Problem: Evaluate the definite integral from 0 to 1 of $(5x^1 + 1)$ dx.

Direct Answer: 3.5

Detailed Work: Antiderivative: integral of $5x^1$ is $5/(2)\ x^2$, and integral of 1 is 1x. Evaluate at 1 and subtract the value at 0: 3.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 191

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 192

Problem: Find the area between y=2 and y=x on the interval $0 \leq x \leq 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 193

Problem: Evaluate the limit: lim as $x \to 9$ of $(x^2 - 81)/(x - 9)$.

Direct Answer: 18

Detailed Work: Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$. For $x \neq 9$, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 194

Problem: Let $f(x) = 2x^3 + 2x^2$. Find f'(2).

Direct Answer: f'(2) = 32

Detailed Work: Use the power rule: $d/dx[2x^3] = 6x^2$, and $d/dx[2x^2] = 4x^1$. Thus $f'(x) = 6x^2 + 4x^1$. Substitute x=2: f'(2) = 32.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 195

Problem: If $f(x)=(2x+2)^3$, find f'(2).

Direct Answer: f'(2) = 216

Detailed Work: Use the chain rule. $f'(x) = 3(2x+2)^2 \times 2 = 6(2x+2)^2$. At x=2, the inside is 6, so $f'(2) = 6(6)^2 = 216$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 196

Problem: A particle has position $s(t) = 2t^3 - 3t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: $v(1) = 3$, $a(1) = 6$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 6t + 3$. Acceleration is $v'(t) = s''(t) = 12t - 6$. Substituting $t=1$ gives $v=3$ and $a=6$.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 197

Problem: Find the absolute minimum value of $f(x) = 2(x - 2)^2 + 6$.

Direct Answer: Minimum value = 6 at $x = 2$

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=2>0$, so it opens upward. The vertex is (2,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 198

Problem: Evaluate the definite integral from 0 to 2 of $(1x^2 + 2)$ dx.

Direct Answer: 6.667

Detailed Work: Antiderivative: integral of $1x^2$ is $1/(3)\ x^3$, and integral of 2 is 2x. Evaluate at 2 and subtract the value at 0: 6.667.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 199

Problem: Solve the differential equation $dy/dx = 2x$ with initial condition $y(0) = 9$.

Direct Answer: $y = (2/2)x^2 + 9$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use $y(0)=9$: $9=0+C$, so $C=9$.

Memory Key: For $dy/dx = g(x)$, integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 200

Problem: Find the area between $y=3$ and $y=x$ on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is $y=3$ and the bottom function is $y=x$ on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3 = 9 - 4.5 = 4.5$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 201

Problem: Find c so that f(x) is continuous at $x = 3$: $f(x)=x^2+c$ for $x<3$, and $f(x)=7x+3$ for $x\ge3$.

Direct Answer: $c = 15$

Detailed Work: For continuity, left value must equal right value at $x=3$. Left: $3^2 + c = 9 + c$. Right: $7(3) + 3 = 24$. Set $9+c = 24$, so $c = 24 - 9 = 15$.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 202

Problem: Let $f(x) = 3x^4 + 3x^3$. Find $f'(3)$.

Direct Answer: $f'(3) = 405$

Detailed Work: Use the power rule: $d/dx[3x^4] = 12x^3$, and $d/dx[3x^3] = 9x^2$. Thus $f'(x) = 12x^3 + 9x^2$. Substitute $x=3$: $f'(3) = 405$.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 203

Problem: For the curve $x^2 + y^2 = 13$, find dy/dx at the point (2, 3).

Direct Answer: $dy/dx = -2/3 = -0.667$

Detailed Work: Differentiate implicitly: $2x + 2y(dy/dx) = 0$. Solve for dy/dx: $dy/dx = -x/y$. At (2,3), $dy/dx = -2/3$.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 204

Problem: A particle has position $s(t) = 3t^3 - 4t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 24, a(2) = 28

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 8t + 4$. Acceleration is $v'(t) = s''(t) = 18t - 8$. Substituting t=2 gives v=24 and a=28.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 205

Problem: Find the absolute minimum value of $f(x) = 3(x - 3)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (3,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 206

Problem: Evaluate the definite integral from 0 to 3 of $(2x^3 + 3)$ dx.

Direct Answer: 49.5

Detailed Work: Antiderivative: integral of $2x^3$ is $2/(4)\ x^4$, and integral of 3 is 3x. Evaluate at 3 and subtract the value at 0: 49.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 207

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 208

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4 = 16 - 8 = 8$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 209

Problem: Evaluate lim as x → 0 of sin(8x)/x.

Direct Answer: 8

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(8x)/x = 8 × [sin(8x)/(8x)]. The bracket approaches 1, so the limit is 8.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 210

Problem: Let $f(x) = 4x^5 + 4x^1$. Find f'(4).

Direct Answer: f'(4) = 5124

Detailed Work: Use the power rule: $d/dx[4x^5] = 20x^4$, and $d/dx[4x^1] = 4x^0$. Thus $f'(x) = 20x^4 + 4x^0$. Substitute x=4: f'(4) = 5124.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 211

Problem: Let $f(x)=e^x + 4x^2$. Find f'(4) exactly.

Direct Answer: $e^4 + 32$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(4)=e^4+32$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 212

Problem: A particle has position $s(t) = 4t^3 - 5t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 83, a(3) = 62

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 10t + 5$. Acceleration is v'(t) = s''(t) = 24t - 10. Substituting t=3 gives v=83 and a=62.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 213

Problem: Find the absolute minimum value of $f(x) = 4(x - 4)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (4,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 214

Problem: Evaluate the definite integral from 0 to 4 of $(3x^4 + 4)$ dx.

Direct Answer: 630.4

Detailed Work: Antiderivative: integral of $3x^4$ is $3/(5)\ x^5$, and integral of 4 is 4x. Evaluate at 4 and subtract the value at 0: 630.4.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 215

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 2.

Direct Answer: $y = (4/2)x^2 + 2$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2$ + C. Use y(0)=2: 2=0+C, so C=2.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 216

Problem: Find the area between y=5 and y=x on the interval 0 ≤ x ≤ 5.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 217

Problem: Evaluate the limit: lim as x → 3 of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For x ≠ 3, cancel (x - 3), leaving x + 3. Substitute x = 3: 3 + 3 = 6.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 218

Problem: Let $f(x) = 5x^2 + 5x^2$. Find f'(1).

Direct Answer: f'(1) = 20

Detailed Work: Use the power rule: $d/dx[5x^2] = 10x^1$, and $d/dx[5x^2] = 10x^1$. Thus $f'(x) = 10x^1 + 10x^1$. Substitute x=1: f'(1) = 20.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 219

Problem: If $f(x)=(5x+1)^3$, find f'(2).

Direct Answer: f'(2) = 1815

Detailed Work: Use the chain rule. $f'(x) = 3(5x+1)^2 \times 5 = 15(5x+1)^2$. At x=2, the inside is 11, so $f'(2) = 15(11)^2 = 1815$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 220

Problem: A particle has position $s(t) = 1t^3 - 6t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 6, a(4) = 12

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 12t + 6$. Acceleration is v'(t) = s''(t) = 6t - 12. Substituting t=4 gives v=6 and a=12.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 221

Problem: Find the absolute minimum value of $f(x) = 1(x - 5)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (5,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 222

Problem: Evaluate the definite integral from 0 to 1 of $(4x^1 + 5)$ dx.

Direct Answer: 7

Detailed Work: Antiderivative: integral of $4x^1$ is $4/(2) x^2$, and integral of 5 is 5x. Evaluate at 1 and subtract the value at 0: 7.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 223

Problem: A quantity satisfies dy/dt = 1y and y(0) = 6. Write y(t).

Direct Answer: $y(t) = 6e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=6, C=6. Thus $y(t)=6e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 224

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 225

Problem: Find c so that f(x) is continuous at x = 6: $f(x)=x^2+c$ for x<6, and f(x)=3x+3 for x≥6.

Direct Answer: c = -15

Detailed Work: For continuity, left value must equal right value at x=6. Left: $6^2 + c = 36 + c$. Right: 3(6) + 3 = 21. Set 36+c = 21, so c = 21 - 36 = -15.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 226

Problem: Let $f(x) = 6x^3 + 6x^3$. Find f'(2).

Direct Answer: f'(2) = 144

Detailed Work: Use the power rule: $d/dx[6x^3] = 18x^2$, and $d/dx[6x^3] = 18x^2$. Thus $f'(x) = 18x^2 + 18x^2$. Substitute x=2: f'(2) = 144.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 227

Problem: For the curve $x^2 + y^2 = 61$, find dy/dx at the point (5, 6).

Direct Answer: dy/dx = -5/6 = -0.833

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (5,6), dy/dx = -5/6.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 228

Problem: A particle has position $s(t) = 2t^3 - 7t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 87, a(5) = 46

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 14t + 7$. Acceleration is v'(t) = s''(t) = 12t - 14. Substituting t=5 gives v=87 and a=46.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 229

Problem: Find the absolute minimum value of $f(x) = 2(x - 6)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (6,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 230

Problem: Evaluate the definite integral from 0 to 2 of $(5x^2 + 6)$ dx.

Direct Answer: 25.333

Detailed Work: Antiderivative: integral of $5x^2$ is $5/(3) x^3$, and integral of 6 is 6x. Evaluate at 2 and subtract the value at 0: 25.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 231

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 4.

Direct Answer: $y = (6/2)x^2 + 4$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2$ + C. Use y(0)=4: 4=0+C, so C=4.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 232

Problem: Find the area between y=3 and y=x on the interval 0 ≤ x ≤ 3.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 233

Problem: Evaluate lim as x → 0 of sin(1x)/x.

Direct Answer: 1

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(1x)/x = 1 × [sin(1x)/(1x)]. The bracket approaches 1, so the limit is 1.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 234

Problem: Let f(x) = $1x^4 + 2x^1$. Find f'(3).

Direct Answer: f'(3) = 110

Detailed Work: Use the power rule: d/dx[$1x^4$] = $4x^3$, and d/dx[$2x^1$] = $2x^0$. Thus f'(x) = $4x^3 + 2x^0$. Substitute x=3: f'(3) = 110.

Memory Key: Power rule: d/dx(x^n) = n x^{n-1}.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 235

Problem: Let f(x)=$e^x + 1x^2$. Find f'(3) exactly.

Direct Answer: $e^3 + 6$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore f'(x)=e^x+2x, so f'(3)=e^3+6.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 236

Problem: A particle has position s(t) = $3t^3 - 2t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 8, a(1) = 14

Detailed Work: Velocity is the derivative of position: v(t) = s'(t) = $9t^2 - 4t + 3$. Acceleration is v'(t) = s''(t) = 18t - 4. Substituting t=1 gives v=8 and a=14.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 237

Problem: Find the absolute minimum value of f(x) = $3(x - 1)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (1,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 238

Problem: Evaluate the definite integral from 0 to 3 of ($1x^3 + 1$) dx.

Direct Answer: 23.25

Detailed Work: Antiderivative: integral of $1x^3$ is 1/(4) x^4, and integral of 1 is 1x. Evaluate at 3 and subtract the value at 0: 23.25.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 239

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: y(t) = $8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus y(t)=$8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 240

Problem: Find the area between y=4 and y=x on the interval $0 \le x \le 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]_0^4 = 16 - 8 = 8$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 241

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel $(x - 6)$, leaving $x + 6$. Substitute $x = 6$: $6 + 6 = 12$.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 242

Problem: Let $f(x) = 2x^5 + 3x^2$. Find f'(4).

Direct Answer: f'(4) = 2584

Detailed Work: Use the power rule: $d/dx[2x^5] = 10x^4$, and $d/dx[3x^2] = 6x^1$. Thus $f'(x) = 10x^4 + 6x^1$. Substitute x=4: f'(4) = 2584.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 243

Problem: If $f(x)=(3x+4)^3$, find f'(2).

Direct Answer: f'(2) = 900

Detailed Work: Use the chain rule. $f'(x) = 3(3x+4)^2 \times 3 = 9(3x+4)^2$. At x=2, the inside is 10, so $f'(2) = 9(10)^2 = 900$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 244

Problem: A particle has position $s(t) = 4t^3 - 3t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 40, a(2) = 42

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 6t + 4$. Acceleration is $v'(t) = s''(t) = 24t - 6$. Substituting t=2 gives v=40 and a=42.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 245

Problem: Find the absolute minimum value of $f(x) = 4(x - 2)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (2,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 246

Problem: Evaluate the definite integral from 0 to 4 of $(2x^4 + 2)$ dx.

Direct Answer: 417.6

Detailed Work: Antiderivative: integral of $2x^4$ is $2/(5)\, x^5$, and integral of 2 is 2x. Evaluate at 4 and subtract the value at 0: 417.6.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 247

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 6.

Direct Answer: $y = (2/2)x^2 + 6$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use y(0)=6: 6=0+C, so C=6.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 248

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 249

Problem: Find c so that f(x) is continuous at x = 1: $f(x)=x^2+c$ for x<1, and f(x)=6x+3 for $x \ge 1$.

Direct Answer: c = 8

Detailed Work: For continuity, left value must equal right value at x=1. Left: $1^2 + c = 1 + c$. Right: 6(1) + 3 = 9. Set 1+c = 9, so c = 9 - 1 = 8.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 250

Problem: Let $f(x) = 3x^2 + 4x^3$. Find f'(1).

Direct Answer: f'(1) = 18

Detailed Work: Use the power rule: $d/dx[3x^2] = 6x^1$, and $d/dx[4x^3] = 12x^2$. Thus $f'(x) = 6x^1 + 12x^2$. Substitute x=1: f'(1) = 18.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 251

Problem: For the curve $x^2 + y^2 = 25$, find dy/dx at the point (3, 4).

Direct Answer: dy/dx = -3/4 = -0.75

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (3,4), dy/dx = -3/4.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 252

Problem: A particle has position $s(t) = 1t^3 - 4t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 8, a(3) = 10

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 8t + 5$. Acceleration is v'(t) = s''(t) = 6t - 8. Substituting t=3 gives v=8 and a=10.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 253

Problem: Find the absolute minimum value of $f(x) = 1(x - 3)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (3,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 254

Problem: Evaluate the definite integral from 0 to 1 of $(3x^1 + 3)$ dx.

Direct Answer: 4.5

Detailed Work: Antiderivative: integral of $3x^1$ is $3/(2) x^2$, and integral of 3 is 3x. Evaluate at 1 and subtract the value at 0: 4.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 255

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 256

Problem: Find the area between y=2 and y=x on the interval $0 \le x \le 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 257

Problem: Evaluate lim as x → 0 of sin(4x)/x.

Direct Answer: 4

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(4x)/x = 4 × [sin(4x)/(4x)]. The bracket approaches 1, so the limit is 4.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 258

Problem: Let $f(x) = 4x^3 + 5x^1$. Find f'(2).

Direct Answer: f'(2) = 53

Detailed Work: Use the power rule: $d/dx[4x^3] = 12x^2$, and $d/dx[5x^1] = 5x^0$. Thus $f'(x) = 12x^2 + 5x^0$. Substitute x=2: f'(2) = 53.

Memory Key: Power rule: $d/dx(x^n) = n\,x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 259

Problem: Let $f(x)=e^x + 4x^2$. Find f'(2) exactly.

Direct Answer: $e^2 + 16$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(2)=e^2+16$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 260

Problem: A particle has position $s(t) = 2t^3 - 5t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 62, a(4) = 38

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 10t + 6$. Acceleration is v'(t) = s''(t) = 12t - 10. Substituting t=4 gives v=62 and a=38.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 261

Problem: Find the absolute minimum value of $f(x) = 2(x - 4)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (4,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 262

Problem: Evaluate the definite integral from 0 to 2 of $(4x^2 + 4)$ dx.

Direct Answer: 18.667

Detailed Work: Antiderivative: integral of $4x^2$ is $4/(3)\ x^3$, and integral of 4 is 4x. Evaluate at 2 and subtract the value at 0: 18.667.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 263

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 8.

Direct Answer: $y = (4/2)x^2 + 8$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2$ + C. Use y(0)=8: 8=0+C, so C=8.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 264

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 265

Problem: Evaluate the limit: lim as $x \to 9$ of $(x^2 - 81)/(x - 9)$.

Direct Answer: 18

Detailed Work: Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$. For $x \neq 9$, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 266

Problem: Let $f(x) = 5x^4 + 6x^2$. Find f'(3).

Direct Answer: f'(3) = 576

Detailed Work: Use the power rule: $d/dx[5x^4] = 20x^3$, and $d/dx[6x^2] = 12x^1$. Thus $f'(x) = 20x^3 + 12x^1$. Substitute x=3: f'(3) = 576.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 267

Problem: If $f(x)=(6x+3)^3$, find f'(2).

Direct Answer: f'(2) = 4050

Detailed Work: Use the chain rule. $f'(x) = 3(6x+3)^2 \times 6 = 18(6x+3)^2$. At x=2, the inside is 15, so $f'(2) = 18(15)^2 = 4050$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 268

Problem: A particle has position $s(t) = 3t^3 - 6t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 172, a(5) = 78

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 12t + 7$. Acceleration is v'(t) = s''(t) = 18t - 12. Substituting t=5 gives v=172 and a=78.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 269

Problem: Find the absolute minimum value of $f(x) = 3(x - 5)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (5,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 270

Problem: Evaluate the definite integral from 0 to 3 of $(5x^3 + 5)$ dx.

Direct Answer: 116.25

Detailed Work: Antiderivative: integral of $5x^3$ is $5/(4)\ x^4$, and integral of 5 is 5x. Evaluate at 3 and subtract the value at 0: 116.25.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 271

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 272

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 273

Problem: Find c so that f(x) is continuous at x = 4: $f(x)=x^2+c$ for x<4, and f(x)=2x+3 for x≥4.

Direct Answer: c = -5

Detailed Work: For continuity, left value must equal right value at x=4. Left: 4^2 + c = 16 + c. Right: 2(4) + 3 = 11. Set 16+c = 11, so c = 11 - 16 = -5.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 274

Problem: Let $f(x) = 6x^5 + 2x^3$. Find f'(4).

Direct Answer: f'(4) = 7776

Detailed Work: Use the power rule: $d/dx[6x^5] = 30x^4$, and $d/dx[2x^3] = 6x^2$. Thus $f'(x) = 30x^4 + 6x^2$. Substitute x=4: f'(4) = 7776.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 275

Problem: For the curve $x^2 + y^2 = 5$, find dy/dx at the point (1, 2).

Direct Answer: dy/dx = -½ = -0.5

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (1,2), dy/dx = -½.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 276

Problem: A particle has position $s(t) = 4t^3 - 7t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 1, a(1) = 10

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 14t + 3$. Acceleration is $v'(t) = s''(t) = 24t - 14$. Substituting t=1 gives v=1 and a=10.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 277

Problem: Find the absolute minimum value of $f(x) = 4(x - 6)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (6,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 278

Problem: Evaluate the definite integral from 0 to 4 of $(1x^4 + 6)$ dx.

Direct Answer: 228.8

Detailed Work: Antiderivative: integral of $1x^4$ is $1/(5)\ x^5$, and integral of 6 is 6x. Evaluate at 4 and subtract the value at 0: 228.8.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 279

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 10.

Direct Answer: $y = (6/2)x^2 + 10$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2 + C$. Use y(0)=10: 10=0+C, so C=10.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 280

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 281

Problem: Evaluate lim as x → 0 of sin(7x)/x.

Direct Answer: 7

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(7x)/x = 7 × [sin(7x)/(7x)]. The bracket approaches 1, so the limit is 7.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 282

Problem: Let $f(x) = 1x^2 + 3x^1$. Find f'(1).

Direct Answer: f'(1) = 5

Detailed Work: Use the power rule: $d/dx[1x^2] = 2x^1$, and $d/dx[3x^1] = 3x^0$. Thus $f'(x) = 2x^1 + 3x^0$. Substitute x=1: f'(1) = 5.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 283

Problem: Let $f(x)=e^x + 1x^2$. Find $f'(1)$ exactly.

Direct Answer: $e^1 + 2$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is $2x$. Therefore $f'(x)=e^x+2x$, so $f'(1)=e^1+2$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes $2x$.

Unit 4: Contextual Applications of Differentiation

Question 284

Problem: A particle has position $s(t) = 1t^3 - 2t^2 + 4t$. Find velocity and acceleration at $t=2$.

Direct Answer: $v(2) = 8$, $a(2) = 8$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 4t + 4$. Acceleration is $v'(t) = s''(t) = 6t - 4$. Substituting $t=2$ gives $v=8$ and $a=8$.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 285

Problem: Find the absolute minimum value of $f(x) = 1(x - 1)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=1>0$, so it opens upward. The vertex is (1,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 286

Problem: Evaluate the definite integral from 0 to 1 of $(2x^1 + 1)$ dx.

Direct Answer: 2

Detailed Work: Antiderivative: integral of $2x^1$ is $2/(2)\ x^2$, and integral of 1 is 1x. Evaluate at 1 and subtract the value at 0: 2.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 287

Problem: A quantity satisfies $dy/dt = 1y$ and $y(0) = 6$. Write $y(t)$.

Direct Answer: $y(t) = 6e^1t$

Detailed Work: The equation $dy/dt = ky$ has exponential solution y = Cekt. Since $y(0)=6$, $C=6$. Thus $y(t)=6e^1t$.

Memory Key: $dy/dt = ky$ gives exponential growth/decay.

Unit 8: Applications of Integration

Question 288

Problem: Find the area between y=2 and y=x on the interval $0 \le x \le 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 289

Problem: Evaluate the limit: lim as x → 3 of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For $x \neq 3$, cancel $(x - 3)$, leaving $x + 3$. Substitute $x = 3$: $3 + 3 = 6$.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 290

Problem: Let $f(x) = 2x^3 + 4x^2$. Find $f'(2)$.

Direct Answer: f'(2) = 40

Detailed Work: Use the power rule: $d/dx[2x^3] = 6x^2$, and $d/dx[4x^2] = 8x^1$. Thus $f'(x) = 6x^2 + 8x^1$. Substitute x=2: f'(2) = 40.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 291

Problem: If $f(x)=(4x+2)^3$, find f'(2).

Direct Answer: f'(2) = 1200

Detailed Work: Use the chain rule. $f'(x) = 3(4x+2)^2 \times 4 = 12(4x+2)^2$. At x=2, the inside is 10, so $f'(2) = 12(10)^2 = 1200$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 292

Problem: A particle has position $s(t) = 2t^3 - 3t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 41, a(3) = 30

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 6t + 5$. Acceleration is v'(t) = s''(t) = 12t - 6. Substituting t=3 gives v=41 and a=30.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 293

Problem: Find the absolute minimum value of $f(x) = 2(x - 2)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (2,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 294

Problem: Evaluate the definite integral from 0 to 2 of $(3x^2 + 2)$ dx.

Direct Answer: 12

Detailed Work: Antiderivative: integral of $3x^2$ is $3/(3)\ x^3$, and integral of 2 is 2x. Evaluate at 2 and subtract the value at 0: 12.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 295

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 3.

Direct Answer: $y = (2/2)x^2 + 3$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use y(0)=3: 3=0+C, so C=3.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 296

Problem: Find the area between y=3 and y=x on the interval 0 ≤ x ≤ 3.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 297

Problem: Find c so that f(x) is continuous at x = 7: $f(x)=x^2+c$ for x<7, and f(x)=5x+3 for x≥7.

Direct Answer: c = -11

Detailed Work: For continuity, left value must equal right value at x=7. Left: $7^2 + c = 49 + c$. Right: 5(7) + 3 = 38. Set 49+c = 38, so c = 38 - 49 = -11.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 298

Problem: Let $f(x) = 3x^4 + 5x^3$. Find f'(3).

Direct Answer: f'(3) = 459

Detailed Work: Use the power rule: $d/dx[3x^4] = 12x^3$, and $d/dx[5x^3] = 15x^2$. Thus $f'(x) = 12x^3 + 15x^2$. Substitute x=3: f'(3) = 459.

Memory Key: Power rule: $d/dx(x^n) = n\, x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 299

Problem: For the curve $x^2 + y^2 = 41$, find dy/dx at the point (4, 5).

Direct Answer: dy/dx = -4/5 = -0.8

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (4,5), dy/dx = -4/5.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 300

Problem: A particle has position $s(t) = 3t^3 - 4t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 118, a(4) = 64

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 8t + 6$. Acceleration is v'(t) = s''(t) = 18t - 8. Substituting t=4 gives v=118 and a=64.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 301

Problem: Find the absolute minimum value of $f(x) = 3(x - 3)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (3,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 302

Problem: Evaluate the definite integral from 0 to 3 of $(4x^3 + 3)$ dx.

Direct Answer: 90

Detailed Work: Antiderivative: integral of $4x^3$ is $4/(4)\, x^4$, and integral of 3 is 3x. Evaluate at 3 and subtract the value at 0: 90.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 303

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: $y(t) = 8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus $y(t)=8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 304

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 305

Problem: Evaluate lim as $x \to 0$ of sin(10x)/x.

Direct Answer: 10

Detailed Work: Use the standard limit lim as $u \to 0$ of sin(u)/u = 1. Write sin(10x)/x = 10 × [sin(10x)/(10x)]. The bracket approaches 1, so the limit is 10.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 306

Problem: Let $f(x) = 4x^5 + 6x^1$. Find f'(4).

Direct Answer: f'(4) = 5126

Detailed Work: Use the power rule: $d/dx[4x^5] = 20x^4$, and $d/dx[6x^1] = 6x^0$. Thus $f'(x) = 20x^4 + 6x^0$. Substitute x=4: f'(4) = 5126.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 307

Problem: Let $f(x)=e^x + 4x^2$. Find f'(4) exactly.

Direct Answer: $e^4 + 32$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(4)=e^4+32$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 308

Problem: A particle has position $s(t) = 4t^3 - 5t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 257, a(5) = 110

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 10t + 7$. Acceleration is v'(t) = s''(t) = 24t - 10. Substituting t=5 gives v=257 and a=110.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 309

Problem: Find the absolute minimum value of $f(x) = 4(x - 4)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (4,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 310

Problem: Evaluate the definite integral from 0 to 4 of $(5x^4 + 4)$ dx.

Direct Answer: 1040

Detailed Work: Antiderivative: integral of $5x^4$ is $5/(5)\ x^5$, and integral of 4 is 4x. Evaluate at 4 and subtract the value at 0: 1040.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 311

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 5.

Direct Answer: $y = (4/2)x^2 + 5$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2$ + C. Use y(0)=5: 5=0+C, so C=5.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 312

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 313

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel (x - 6), leaving x + 6. Substitute x = 6: 6 + 6 = 12.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 314

Problem: Let $f(x) = 5x^2 + 2x^2$. Find f'(1).

Direct Answer: f'(1) = 14

Detailed Work: Use the power rule: $d/dx[5x^2] = 10x^1$, and $d/dx[2x^2] = 4x^1$. Thus $f'(x) = 10x^1 + 4x^1$. Substitute x=1: f'(1) = 14.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 315

Problem: If $f(x)=(2x+1)^3$, find f'(2).

Direct Answer: f'(2) = 150

Detailed Work: Use the chain rule. $f'(x) = 3(2x+1)^2 \times 2 = 6(2x+1)^2$. At x=2, the inside is 5, so $f'(2) = 6(5)^2 = 150$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 316

Problem: A particle has position $s(t) = 1t^3 - 6t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = -6, a(1) = -6

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 12t + 3$. Acceleration is v'(t) = s''(t) = 6t - 12. Substituting t=1 gives v=-6 and a=-6.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 317

Problem: Find the absolute minimum value of $f(x) = 1(x - 5)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (5,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 318

Problem: Evaluate the definite integral from 0 to 1 of $(1x^1 + 5)$ dx.

Direct Answer: 5.5

Detailed Work: Antiderivative: integral of $1x^1$ is $1/(2) x^2$, and integral of 5 is 5x. Evaluate at 1 and subtract the value at 0: 5.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 319

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^{1}t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^{1}t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 320

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2$ = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 321

Problem: Find c so that f(x) is continuous at x = 2: $f(x)=x^2+c$ for x<2, and f(x)=8x+3 for x≥2.

Direct Answer: c = 15

Detailed Work: For continuity, left value must equal right value at x=2. Left: 2^2 + c = 4 + c. Right: 8(2) + 3 = 19. Set 4+c = 19, so c = 19 - 4 = 15.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 322

Problem: Let $f(x) = 6x^3 + 3x^3$. Find f'(2).

Direct Answer: f'(2) = 108

Detailed Work: Use the power rule: $d/dx[6x^3] = 18x^2$, and $d/dx[3x^3] = 9x^2$. Thus $f'(x) = 18x^2 + 9x^2$. Substitute x=2: f'(2) = 108.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 323

Problem: For the curve $x^2 + y^2 = 13$, find dy/dx at the point (2, 3).

Direct Answer: dy/dx = -2/3 = -0.667

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (2,3), dy/dx = -2/3.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 324

Problem: A particle has position $s(t) = 2t^3 - 7t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 0, a(2) = 10

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 14t + 4$. Acceleration is v'(t) = s''(t) = 12t - 14. Substituting t=2 gives v=0 and a=10.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 325

Problem: Find the absolute minimum value of $f(x) = 2(x - 6)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (6,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 326

Problem: Evaluate the definite integral from 0 to 2 of $(2x^2 + 6)$ dx.

Direct Answer: 17.333

Detailed Work: Antiderivative: integral of $2x^2$ is $2/(3)\ x^3$, and integral of 6 is 6x. Evaluate at 2 and subtract the value at 0: 17.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 327

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 7.

Direct Answer: $y = (6/2)x^2 + 7$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2 + C$. Use y(0)=7: 7=0+C, so C=7.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 328

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 329

Problem: Evaluate lim as x → 0 of sin(3x)/x.

Direct Answer: 3

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(3x)/x = 3 × [sin(3x)/(3x)]. The bracket approaches 1, so the limit is 3.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 330

Problem: Let $f(x) = 1x^4 + 4x^1$. Find f'(3).

Direct Answer: f'(3) = 112

Detailed Work: Use the power rule: $d/dx[1x^4] = 4x^3$, and $d/dx[4x^1] = 4x^0$. Thus $f'(x) = 4x^3 + 4x^0$. Substitute x=3: f'(3) = 112.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 331

Problem: Let $f(x)=e^x + 1x^2$. Find f'(3) exactly.

Direct Answer: $e^3 + 6$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore $f'(x)=e^x+2x$, so $f'(3)=e^3+6$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 332

Problem: A particle has position $s(t) = 3t^3 - 2t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 74, a(3) = 50

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 4t + 5$. Acceleration is v'(t) = s''(t) = 18t - 4. Substituting t=3 gives v=74 and a=50.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 333

Problem: Find the absolute minimum value of $f(x) = 3(x - 1)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (1,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 334

Problem: Evaluate the definite integral from 0 to 3 of $(3x^3 + 1)$ dx.

Direct Answer: 63.75

Detailed Work: Antiderivative: integral of $3x^3$ is $3/(4)\ x^4$, and integral of 1 is 1x. Evaluate at 3 and subtract the value at 0: 63.75.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 335

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 336

Problem: Find the area between y=4 and y=x on the interval $0 \leq x \leq 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 337

Problem: Evaluate the limit: lim as $x \to 9$ of $(x^2 - 81)/(x - 9)$.

Direct Answer: 18

Detailed Work: Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$. For $x \neq 9$, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 338

Problem: Let $f(x) = 2x^5 + 5x^2$. Find f'(4).

Direct Answer: f'(4) = 2600

Detailed Work: Use the power rule: $d/dx[2x^5] = 10x^4$, and $d/dx[5x^2] = 10x^1$. Thus $f'(x) = 10x^4 + 10x^1$. Substitute x=4: f'(4) = 2600.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 339

Problem: If $f(x)=(5x+4)^3$, find f'(2).

Direct Answer: f'(2) = 2940

Detailed Work: Use the chain rule. $f'(x) = 3(5x+4)^2 \times 5 = 15(5x+4)^2$. At x=2, the inside is 14, so $f'(2) = 15(14)^2 = 2940$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 340

Problem: A particle has position $s(t) = 4t^3 - 3t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 174, a(4) = 90

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 6t + 6$. Acceleration is v'(t) = s''(t) = 24t - 6. Substituting t=4 gives v=174 and a=90.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 341

Problem: Find the absolute minimum value of $f(x) = 4(x - 2)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (2,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 342

Problem: Evaluate the definite integral from 0 to 4 of $(4x^4 + 2)$ dx.

Direct Answer: 827.2

Detailed Work: Antiderivative: integral of $4x^4$ is $4/(5)\ x^5$, and integral of 2 is 2x. Evaluate at 4 and subtract the value at 0: 827.2.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 343

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 9.

Direct Answer: $y = (2/2)x^2 + 9$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2$ + C. Use y(0)=9: 9=0+C, so C=9.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 344

Problem: Find the area between y=5 and y=x on the interval 0 ≤ x ≤ 5.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 345

Problem: Find c so that f(x) is continuous at x = 5: $f(x)=x^2+c$ for x<5, and f(x)=4x+3 for x≥5.

Direct Answer: c = -2

Detailed Work: For continuity, left value must equal right value at x=5. Left: 5^2 + c = 25 + c. Right: 4(5) + 3 = 23. Set 25+c = 23, so c = 23 - 25 = -2.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 346

Problem: Let $f(x) = 3x^2 + 6x^3$. Find f'(1).

Direct Answer: f'(1) = 24

Detailed Work: Use the power rule: $d/dx[3x^2] = 6x^1$, and $d/dx[6x^3] = 18x^2$. Thus $f'(x) = 6x^1 + 18x^2$. Substitute x=1: f'(1) = 24.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 347

Problem: For the curve $x^2 + y^2 = 61$, find dy/dx at the point (5, 6).

Direct Answer: dy/dx = -5/6 = -0.833

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (5,6), dy/dx = -5/6.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 348

Problem: A particle has position $s(t) = 1t^3 - 4t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 42, a(5) = 22

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 8t + 7$. Acceleration is v'(t) = s''(t) = 6t - 8. Substituting t=5 gives v=42 and a=22.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 349

Problem: Find the absolute minimum value of $f(x) = 1(x - 3)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (3,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 350

Problem: Evaluate the definite integral from 0 to 1 of $(5x^1 + 3)$ dx.

Direct Answer: 5.5

Detailed Work: Antiderivative: integral of $5x^1$ is $5/(2) x^2$, and integral of 3 is 3x. Evaluate at 1 and subtract the value at 0: 5.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 351

Problem: A quantity satisfies dy/dt = 1y and y(0) = 6. Write y(t).

Direct Answer: $y(t) = 6e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=6, C=6. Thus $y(t)=6e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 352

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2$ = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 353

Problem: Evaluate lim as x → 0 of sin(6x)/x.

Direct Answer: 6

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(6x)/x = 6 × [sin(6x)/(6x)]. The bracket approaches 1, so the limit is 6.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 354

Problem: Let $f(x) = 4x^3 + 2x^1$. Find f'(2).

Direct Answer: f'(2) = 50

Detailed Work: Use the power rule: $d/dx[4x^3] = 12x^2$, and $d/dx[2x^1] = 2x^0$. Thus $f'(x) = 12x^2 + 2x^0$. Substitute x=2: f'(2) = 50.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 355

Problem: Let $f(x)=e^x + 4x^2$. Find f'(2) exactly.

Direct Answer: $e^2 + 16$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(2)=e^2+16$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 356

Problem: A particle has position $s(t) = 2t^3 - 5t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = -1, a(1) = 2

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 10t + 3$. Acceleration is v'(t) = s''(t) = 12t - 10. Substituting t=1 gives v=-1 and a=2.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 357

Problem: Find the absolute minimum value of $f(x) = 2(x - 4)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (4,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 358

Problem: Evaluate the definite integral from 0 to 2 of $(1x^2 + 4)$ dx.

Direct Answer: 10.667

Detailed Work: Antiderivative: integral of $1x^2$ is 1/(3) x^3, and integral of 4 is 4x. Evaluate at 2 and subtract the value at 0: 10.667.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 359

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 2.

Direct Answer: $y = (4/2)x^2 + 2$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2$ + C. Use y(0)=2: 2=0+C, so C=2.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 360

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 361

Problem: Evaluate the limit: lim as x → 3 of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For $x \neq 3$, cancel (x - 3), leaving x + 3. Substitute x = 3: 3 + 3 = 6.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 362

Problem: Let $f(x) = 5x^4 + 3x^2$. Find f'(3).

Direct Answer: f'(3) = 558

Detailed Work: Use the power rule: $d/dx[5x^4] = 20x^3$, and $d/dx[3x^2] = 6x^1$. Thus $f'(x) = 20x^3 + 6x^1$. Substitute x=3: f'(3) = 558.

Memory Key: Power rule: $d/dx(x^n) = n\, x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 363

Problem: If $f(x)=(3x+3)^3$, find f'(2).

Direct Answer: f'(2) = 729

Detailed Work: Use the chain rule. $f'(x) = 3(3x+3)^2 \times 3 = 9(3x+3)^2$. At x=2, the inside is 9, so $f'(2) = 9(9)^2 = 729$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 364

Problem: A particle has position $s(t) = 3t^3 - 6t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 16, a(2) = 24

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 12t + 4$. Acceleration is v'(t) = s''(t) = 18t - 12. Substituting t=2 gives v=16 and a=24.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 365

Problem: Find the absolute minimum value of $f(x) = 3(x - 5)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (5,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 366

Problem: Evaluate the definite integral from 0 to 3 of $(2x^3 + 5)$ dx.

Direct Answer: 55.5

Detailed Work: Antiderivative: integral of $2x^3$ is $2/(4)\, x^4$, and integral of 5 is 5x. Evaluate at 3 and subtract the value at 0: 55.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 367

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: $y(t) = 8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus $y(t)=8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 368

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 369

Problem: Find c so that f(x) is continuous at x = 8: $f(x)=x^2+c$ for x<8, and f(x)=7x+3 for x≥8.

Direct Answer: c = -5

Detailed Work: For continuity, left value must equal right value at x=8. Left: $8^2 + c = 64 + c$. Right: 7(8) + 3 = 59. Set 64+c = 59, so c = 59 - 64 = -5.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 370

Problem: Let $f(x) = 6x^5 + 4x^3$. Find f'(4).

Direct Answer: f'(4) = 7872

Detailed Work: Use the power rule: $d/dx[6x^5] = 30x^4$, and $d/dx[4x^3] = 12x^2$. Thus $f'(x) = 30x^4 + 12x^2$. Substitute x=4: f'(4) = 7872.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 371

Problem: For the curve $x^2 + y^2 = 25$, find dy/dx at the point (3, 4).

Direct Answer: dy/dx = -3/4 = -0.75

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (3,4), dy/dx = -3/4.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 372

Problem: A particle has position $s(t) = 4t^3 - 7t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 71, a(3) = 58

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 14t + 5$. Acceleration is v'(t) = s''(t) = 24t - 14. Substituting t=3 gives v=71 and a=58.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 373

Problem: Find the absolute minimum value of $f(x) = 4(x - 6)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (6,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 374

Problem: Evaluate the definite integral from 0 to 4 of $(3x^4 + 6)$ dx.

Direct Answer: 638.4

Detailed Work: Antiderivative: integral of $3x^4$ is $3/(5) x^5$, and integral of 6 is 6x. Evaluate at 4 and subtract the value at 0: 638.4.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 375

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 4.

Direct Answer: $y = (6/2)x^2 + 4$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2$ + C. Use y(0)=4: 4=0+C, so C=4.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 376

Problem: Find the area between y=5 and y=x on the interval 0 ≤ x ≤ 5.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 377

Problem: Evaluate lim as $x \to 0$ of sin(9x)/x.

Direct Answer: 9

Detailed Work: Use the standard limit lim as $u \to 0$ of sin(u)/u = 1. Write sin(9x)/x = 9 × [sin(9x)/(9x)]. The bracket approaches 1, so the limit is 9.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 378

Problem: Let $f(x) = 1x^2 + 5x^1$. Find f'(1).

Direct Answer: f'(1) = 7

Detailed Work: Use the power rule: $d/dx[1x^2] = 2x^1$, and $d/dx[5x^1] = 5x^0$. Thus $f'(x) = 2x^1 + 5x^0$. Substitute x=1: f'(1) = 7.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 379

Problem: Let $f(x)=e^x + 1x^2$. Find f'(1) exactly.

Direct Answer: $e^1 + 2$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore $f'(x)=e^x+2x$, so $f'(1)=e^1+2$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 380

Problem: A particle has position $s(t) = 1t^3 - 2t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 38, a(4) = 20

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 4t + 6$. Acceleration is v'(t) = s''(t) = 6t - 4. Substituting t=4 gives v=38 and a=20.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 381

Problem: Find the absolute minimum value of $f(x) = 1(x - 1)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (1,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 382

Problem: Evaluate the definite integral from 0 to 1 of $(4x^1 + 1)$ dx.

Direct Answer: 3

Detailed Work: Antiderivative: integral of $4x^1$ is $4/(2)\ x^2$, and integral of 1 is 1x. Evaluate at 1 and subtract the value at 0: 3.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 383

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 384

Problem: Find the area between y=2 and y=x on the interval $0 \le x \le 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 385

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel (x - 6), leaving x + 6. Substitute x = 6: 6 + 6 = 12.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 386

Problem: Let $f(x) = 2x^3 + 6x^2$. Find f'(2).

Direct Answer: f'(2) = 48

Detailed Work: Use the power rule: $d/dx[2x^3] = 6x^2$, and $d/dx[6x^2] = 12x^1$. Thus $f'(x) = 6x^2 + 12x^1$. Substitute x=2: f'(2) = 48.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 387

Problem: If $f(x)=(6x+2)^3$, find f'(2).

Direct Answer: f'(2) = 3528

Detailed Work: Use the chain rule. $f'(x) = 3(6x+2)^2 \times 6 = 18(6x+2)^2$. At x=2, the inside is 14, so $f'(2) = 18(14)^2 = 3528$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 388

Problem: A particle has position $s(t) = 2t^3 - 3t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 127, a(5) = 54

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 6t + 7$. Acceleration is v'(t) = s''(t) = 12t - 6. Substituting t=5 gives v=127 and a=54.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 389

Problem: Find the absolute minimum value of $f(x) = 2(x - 2)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (2,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 390

Problem: Evaluate the definite integral from 0 to 2 of $(5x^2 + 2)$ dx.

Direct Answer: 17.333

Detailed Work: Antiderivative: integral of $5x^2$ is $5/(3) x^3$, and integral of 2 is 2x. Evaluate at 2 and subtract the value at 0: 17.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 391

Problem: Solve the differential equation $dy/dx = 2x$ with initial condition $y(0) = 6$.

Direct Answer: $y = (2/2)x^2 + 6$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use y(0)=6: 6=0+C, so C=6.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 392

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]_0^3 = 9 - 4.5 = 4.5$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 393

Problem: Find c so that f(x) is continuous at x = 3: $f(x)=x^2+c$ for $x<3$, and $f(x)=3x+3$ for $x\ge3$.

Direct Answer: c = 3

Detailed Work: For continuity, left value must equal right value at x=3. Left: $3^2 + c = 9 + c$. Right: 3(3) + 3 = 12. Set 9+c = 12, so c = 12 - 9 = 3.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 394

Problem: Let $f(x) = 3x^4 + 2x^3$. Find f'(3).

Direct Answer: f'(3) = 378

Detailed Work: Use the power rule: $d/dx[3x^4] = 12x^3$, and $d/dx[2x^3] = 6x^2$. Thus $f'(x) = 12x^3 + 6x^2$. Substitute x=3: f'(3) = 378.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 395

Problem: For the curve $x^2 + y^2 = 5$, find dy/dx at the point (1, 2).

Direct Answer: dy/dx = -½ = -0.5

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (1,2), dy/dx = -½.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 396

Problem: A particle has position $s(t) = 3t^3 - 4t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 4, a(1) = 10

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 8t + 3$. Acceleration is v'(t) = s''(t) = 18t - 8. Substituting t=1 gives v=4 and a=10.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 397

Problem: Find the absolute minimum value of $f(x) = 3(x - 3)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (3,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 398

Problem: Evaluate the definite integral from 0 to 3 of $(1x^3 + 3)$ dx.

Direct Answer: 29.25

Detailed Work: Antiderivative: integral of $1x^3$ is $1/(4)\ x^4$, and integral of 3 is 3x. Evaluate at 3 and subtract the value at 0: 29.25.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 399

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 400

Problem: Find the area between y=4 and y=x on the interval 0 ≤ x ≤ 4.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = [4x - x^2/2]_0^4 = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 401

Problem: Evaluate lim as x → 0 of sin(2x)/x.

Direct Answer: 2

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(2x)/x = 2 × [sin(2x)/(2x)]. The bracket approaches 1, so the limit is 2.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 402

Problem: Let $f(x) = 4x^5 + 3x^1$. Find f'(4).

Direct Answer: f'(4) = 5123

Detailed Work: Use the power rule: $d/dx[4x^5] = 20x^4$, and $d/dx[3x^1] = 3x^0$. Thus $f'(x) = 20x^4 + 3x^0$. Substitute x=4: f'(4) = 5123.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 403

Problem: Let $f(x)=e^x + 4x^2$. Find f'(4) exactly.

Direct Answer: $e^4 + 32$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(4)=e^4+32$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 404

Problem: A particle has position $s(t) = 4t^3 - 5t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 32, a(2) = 38

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 10t + 4$. Acceleration is v'(t) = s''(t) = 24t - 10. Substituting t=2 gives v=32 and a=38.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 405

Problem: Find the absolute minimum value of $f(x) = 4(x - 4)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (4,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 406

Problem: Evaluate the definite integral from 0 to 4 of $(2x^4 + 4)$ dx.

Direct Answer: 425.6

Detailed Work: Antiderivative: integral of $2x^4$ is $2/(5)\ x^5$, and integral of 4 is 4x. Evaluate at 4 and subtract the value at 0: 425.6.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 407

Problem: Solve the differential equation dy/dx = 4x with initial condition y(0) = 8.

Direct Answer: $y = (4/2)x^2 + 8$

Detailed Work: Integrate both sides with respect to x: y = integral 4x dx = $(4/2)x^2$ + C. Use y(0)=8: 8=0+C, so C=8.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 408

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 409

Problem: Evaluate the limit: lim as $x \to 9$ of $(x^2 - 81)/(x - 9)$.

Direct Answer: 18

Detailed Work: Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$. For $x \neq 9$, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 410

Problem: Let $f(x) = 5x^2 + 4x^2$. Find f'(1).

Direct Answer: f'(1) = 18

Detailed Work: Use the power rule: $d/dx[5x^2] = 10x^1$, and $d/dx[4x^2] = 8x^1$. Thus $f'(x) = 10x^1 + 8x^1$. Substitute x=1: f'(1) = 18.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 411

Problem: If $f(x)=(4x+1)^3$, find f'(2).

Direct Answer: f'(2) = 972

Detailed Work: Use the chain rule. $f'(x) = 3(4x+1)^2 \times 4 = 12(4x+1)^2$. At x=2, the inside is 9, so $f'(2) = 12(9)^2 = 972$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 412

Problem: A particle has position $s(t) = 1t^3 - 6t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = -4, a(3) = 6

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 12t + 5$. Acceleration is v'(t) = s''(t) = 6t - 12. Substituting t=3 gives v=-4 and a=6.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 413

Problem: Find the absolute minimum value of $f(x) = 1(x - 5)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=1>0$, so it opens upward. The vertex is (5,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 414

Problem: Evaluate the definite integral from 0 to 1 of $(3x^1 + 5)$ dx.

Direct Answer: 6.5

Detailed Work: Antiderivative: integral of $3x^1$ is $3/(2)\ x^2$, and integral of 5 is 5x. Evaluate at 1 and subtract the value at 0: 6.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 415

Problem: A quantity satisfies dy/dt = 1y and y(0) = 6. Write y(t).

Direct Answer: $y(t) = 6e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=6, C=6. Thus $y(t)=6e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 416

Problem: Find the area between y=2 and y=x on the interval $0 \leq x \leq 2$.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 417

Problem: Find c so that f(x) is continuous at x = 6: $f(x)=x^2+c$ for x<6, and f(x)=6x+3 for x≥6.

Direct Answer: c = 3

Detailed Work: For continuity, left value must equal right value at x=6. Left: $6^2 + c = 36 + c$. Right: 6(6) + 3 = 39. Set 36+c = 39, so c = 39 - 36 = 3.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 418

Problem: Let $f(x) = 6x^3 + 5x^3$. Find f'(2).

Direct Answer: f'(2) = 132

Detailed Work: Use the power rule: $d/dx[6x^3] = 18x^2$, and $d/dx[5x^3] = 15x^2$. Thus $f'(x) = 18x^2 + 15x^2$. Substitute x=2: f'(2) = 132.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 419

Problem: For the curve $x^2 + y^2 = 41$, find dy/dx at the point (4, 5).

Direct Answer: dy/dx = -4/5 = -0.8

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (4,5), dy/dx = -4/5.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 420

Problem: A particle has position $s(t) = 2t^3 - 7t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 46, a(4) = 34

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 14t + 6$. Acceleration is $v'(t) = s''(t) = 12t - 14$. Substituting t=4 gives v=46 and a=34.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 421

Problem: Find the absolute minimum value of $f(x) = 2(x - 6)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (6,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 422

Problem: Evaluate the definite integral from 0 to 2 of $(4x^2 + 6)$ dx.

Direct Answer: 22.667

Detailed Work: Antiderivative: integral of $4x^2$ is $4/(3)\ x^3$, and integral of 6 is 6x. Evaluate at 2 and subtract the value at 0: 22.667.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 423

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 10.

Direct Answer: $y = (6/2)x^2 + 10$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2$ + C. Use y(0)=10: 10=0+C, so C=10.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 424

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 425

Problem: Evaluate lim as x → 0 of sin(5x)/x.

Direct Answer: 5

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(5x)/x = 5 × [sin(5x)/(5x)]. The bracket approaches 1, so the limit is 5.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 426

Problem: Let $f(x) = 1x^4 + 6x^1$. Find f'(3).

Direct Answer: f'(3) = 114

Detailed Work: Use the power rule: $d/dx[1x^4] = 4x^3$, and $d/dx[6x^1] = 6x^0$. Thus $f'(x) = 4x^3 + 6x^0$. Substitute x=3: f'(3) = 114.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 427

Problem: Let $f(x)=e^x + 1x^2$. Find f'(3) exactly.

Direct Answer: $e^3 + 6$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore $f'(x)=e^x+2x$, so $f'(3)=e^3+6$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 428

Problem: A particle has position $s(t) = 3t^3 - 2t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 212, a(5) = 86

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 4t + 7$. Acceleration is $v'(t) = s''(t) = 18t - 4$. Substituting t=5 gives v=212 and a=86.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 429

Problem: Find the absolute minimum value of $f(x) = 3(x - 1)^2 + 7$.

Direct Answer: Minimum value = 7 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (1,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 430

Problem: Evaluate the definite integral from 0 to 3 of $(5x^3 + 1)$ dx.

Direct Answer: 104.25

Detailed Work: Antiderivative: integral of $5x^3$ is $5/(4)\ x^4$, and integral of 1 is 1x. Evaluate at 3 and subtract the value at 0: 104.25.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 431

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: $y(t) = 8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus $y(t)=8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 432

Problem: Find the area between y=4 and y=x on the interval $0 \le x \le 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4 = 16 - 8 = 8$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 433

Problem: Evaluate the limit: lim as x → 3 of $(x^2 - 9)/(x - 3)$.

Direct Answer: 6

Detailed Work: Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. For $x \neq 3$, cancel (x - 3), leaving x + 3. Substitute x = 3: 3 + 3 = 6.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 434

Problem: Let $f(x) = 2x^5 + 2x^2$. Find f'(4).

Direct Answer: f'(4) = 2576

Detailed Work: Use the power rule: $d/dx[2x^5] = 10x^4$, and $d/dx[2x^2] = 4x^1$. Thus $f'(x) = 10x^4 + 4x^1$. Substitute x=4: f'(4) = 2576.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 435

Problem: If $f(x)=(2x+4)^3$, find f'(2).

Direct Answer: f'(2) = 384

Detailed Work: Use the chain rule. $f'(x) = 3(2x+4)^2 \times 2 = 6(2x+4)^2$. At x=2, the inside is 8, so $f'(2) = 6(8)^2 = 384$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 436

Problem: A particle has position $s(t) = 4t^3 - 3t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 9, a(1) = 18

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 6t + 3$. Acceleration is $v'(t) = s''(t) = 24t - 6$. Substituting t=1 gives v=9 and a=18.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 437

Problem: Find the absolute minimum value of $f(x) = 4(x - 2)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (2,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 438

Problem: Evaluate the definite integral from 0 to 4 of $(1x^4 + 2)$ dx.

Direct Answer: 212.8

Detailed Work: Antiderivative: integral of $1x^4$ is $1/(5)\ x^5$, and integral of 2 is 2x. Evaluate at 4 and subtract the value at 0: 212.8.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 439

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 3.

Direct Answer: $y = (2/2)x^2 + 3$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2 + C$. Use y(0)=3: 3=0+C, so C=3.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 440

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 441

Problem: Find c so that f(x) is continuous at x = 1: $f(x)=x^2+c$ for x<1, and f(x)=2x+3 for $x \ge 1$.

Direct Answer: c = 4

Detailed Work: For continuity, left value must equal right value at x=1. Left: $1^2 + c = 1 + c$. Right: 2(1) + 3 = 5. Set 1+c = 5, so c = 5 - 1 = 4.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 442

Problem: Let $f(x) = 3x^2 + 3x^3$. Find f'(1).

Direct Answer: f'(1) = 15

Detailed Work: Use the power rule: $d/dx[3x^2] = 6x^1$, and $d/dx[3x^3] = 9x^2$. Thus $f'(x) = 6x^1 + 9x^2$. Substitute x=1: f'(1) = 15.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 443

Problem: For the curve $x^2 + y^2 = 13$, find dy/dx at the point (2, 3).

Direct Answer: dy/dx = -2/3 = -0.667

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (2,3), dy/dx = -2/3.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 444

Problem: A particle has position $s(t) = 1t^3 - 4t^2 + 4t$. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 0, a(2) = 4

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 8t + 4$. Acceleration is v'(t) = s''(t) = 6t - 8. Substituting t=2 gives v=0 and a=4.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 445

Problem: Find the absolute minimum value of $f(x) = 1(x - 3)^2 + 2$.

Direct Answer: Minimum value = 2 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (3,2), and the minimum value is the y-value 2.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 446

Problem: Evaluate the definite integral from 0 to 1 of $(2x^1 + 3)$ dx.

Direct Answer: 4

Detailed Work: Antiderivative: integral of $2x^1$ is $2/(2) x^2$, and integral of 3 is 3x. Evaluate at 1 and subtract the value at 0: 4.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 447

Problem: A quantity satisfies dy/dt = 1y and y(0) = 2. Write y(t).

Direct Answer: $y(t) = 2e^1t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=2, C=2. Thus $y(t)=2e^1t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 448

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = $[2x - x^2/2]\_0^2 = 4 - 2 = 2$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 449

Problem: Evaluate lim as $x \to 0$ of $\sin(8x)/x$.

Direct Answer: 8

Detailed Work: Use the standard limit lim as $u \to 0$ of $\sin(u)/u = 1$. Write $\sin(8x)/x = 8 \times [\sin(8x)/(8x)]$. The bracket approaches 1, so the limit is 8.

Memory Key: Convert trig limits to the standard form $\sin(u)/u$.

Unit 2: Differentiation - Definition and Basic Rules

Question 450

Problem: Let $f(x) = 4x^3 + 4x^1$. Find $f'(2)$.

Direct Answer: $f'(2) = 52$

Detailed Work: Use the power rule: $d/dx[4x^3] = 12x^2$, and $d/dx[4x^1] = 4x^0$. Thus $f'(x) = 12x^2 + 4x^0$. Substitute x=2: $f'(2) = 52$.

Memory Key: Power rule: $d/dx(x^n) = n\, x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 451

Problem: Let $f(x)=e^x + 4x^2$. Find $f'(2)$ exactly.

Direct Answer: $e^2 + 16$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is $8x$. Therefore $f'(x)=e^x+8x$, so $f'(2)=e^2+16$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes $2x$.

Unit 4: Contextual Applications of Differentiation

Question 452

Problem: A particle has position $s(t) = 2t^3 - 5t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: $v(3) = 29$, $a(3) = 26$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 6t^2 - 10t + 5$. Acceleration is $v'(t) = s''(t) = 12t - 10$. Substituting t=3 gives v=29 and a=26.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 453

Problem: Find the absolute minimum value of $f(x) = 2(x - 4)^2 + 3$.

Direct Answer: Minimum value = 3 at x = 4

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with $a=2>0$, so it opens upward. The vertex is (4,3), and the minimum value is the y-value 3.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 454

Problem: Evaluate the definite integral from 0 to 2 of $(3x^2 + 4)\,dx$.

Direct Answer: 16

Detailed Work: Antiderivative: integral of $3x^2$ is $3/(3)\, x^3$, and integral of 4 is 4x. Evaluate at 2 and subtract the value at 0: 16.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 455

Problem: Solve the differential equation $dy/dx = 4x$ with initial condition $y(0) = 5$.

Direct Answer: $y = (4/2)x^2 + 5$

Detailed Work: Integrate both sides with respect to x: $y = \text{integral } 4x\, dx = (4/2)x^2 + C$. Use $y(0)=5$: $5=0+C$, so $C=5$.

Memory Key: For $dy/dx = g(x)$, integrate $g(x)$ and use the initial condition.

Unit 8: Applications of Integration

Question 456

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 457

Problem: Evaluate the limit: lim as $x \to 6$ of $(x^2 - 36)/(x - 6)$.

Direct Answer: 12

Detailed Work: Factor the numerator: $x^2 - 36 = (x - 6)(x + 6)$. For $x \neq 6$, cancel (x - 6), leaving x + 6. Substitute x = 6: 6 + 6 = 12.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 458

Problem: Let $f(x) = 5x^4 + 5x^2$. Find f'(3).

Direct Answer: f'(3) = 570

Detailed Work: Use the power rule: $d/dx[5x^4] = 20x^3$, and $d/dx[5x^2] = 10x^1$. Thus $f'(x) = 20x^3 + 10x^1$. Substitute x=3: f'(3) = 570.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 459

Problem: If $f(x)=(5x+3)^3$, find f'(2).

Direct Answer: f'(2) = 2535

Detailed Work: Use the chain rule. $f'(x) = 3(5x+3)^2 \times 5 = 15(5x+3)^2$. At x=2, the inside is 13, so $f'(2) = 15(13)^2 = 2535$.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 460

Problem: A particle has position $s(t) = 3t^3 - 6t^2 + 6t$. Find velocity and acceleration at t=4.

Direct Answer: v(4) = 102, a(4) = 60

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 12t + 6$. Acceleration is $v'(t) = s''(t) = 18t - 12$. Substituting t=4 gives v=102 and a=60.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 461

Problem: Find the absolute minimum value of $f(x) = 3(x - 5)^2 + 4$.

Direct Answer: Minimum value = 4 at x = 5

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (5,4), and the minimum value is the y-value 4.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 462

Problem: Evaluate the definite integral from 0 to 3 of $(4x^3 + 5)$ dx.

Direct Answer: 96

Detailed Work: Antiderivative: integral of $4x^3$ is $4/(4)\ x^4$, and integral of 5 is 5x. Evaluate at 3 and subtract the value at 0: 96.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 463

Problem: A quantity satisfies dy/dt = 3y and y(0) = 4. Write y(t).

Direct Answer: $y(t) = 4e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=4, C=4. Thus $y(t)=4e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 464

Problem: Find the area between y=4 and y=x on the interval $0 \le x \le 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]_0^4$ = 16 - 8 = 8.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 465

Problem: Find c so that f(x) is continuous at x = 4: $f(x)=x^2+c$ for x<4, and f(x)=5x+3 for $x \ge 4$.

Direct Answer: c = 7

Detailed Work: For continuity, left value must equal right value at x=4. Left: $4^2 + c = 16 + c$. Right: 5(4) + 3 = 23. Set 16+c = 23, so c = 23 - 16 = 7.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 466

Problem: Let $f(x) = 6x^5 + 6x^3$. Find f'(4).

Direct Answer: f'(4) = 7968

Detailed Work: Use the power rule: $d/dx[6x^5] = 30x^4$, and $d/dx[6x^3] = 18x^2$. Thus $f'(x) = 30x^4 + 18x^2$. Substitute x=4: f'(4) = 7968.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 467

Problem: For the curve $x^2 + y^2 = 61$, find dy/dx at the point (5, 6).

Direct Answer: dy/dx = -5/6 = -0.833

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (5,6), dy/dx = -5/6.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 468

Problem: A particle has position $s(t) = 4t^3 - 7t^2 + 7t$. Find velocity and acceleration at t=5.

Direct Answer: v(5) = 237, a(5) = 106

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 14t + 7$. Acceleration is v'(t) = s''(t) = 24t - 14. Substituting t=5 gives v=237 and a=106.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 469

Problem: Find the absolute minimum value of $f(x) = 4(x - 6)^2 + 5$.

Direct Answer: Minimum value = 5 at x = 6

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=4>0, so it opens upward. The vertex is (6,5), and the minimum value is the y-value 5.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 470

Problem: Evaluate the definite integral from 0 to 4 of $(5x^4 + 6)$ dx.

Direct Answer: 1048

Detailed Work: Antiderivative: integral of $5x^4$ is $5/(5)\ x^5$, and integral of 6 is 6x. Evaluate at 4 and subtract the value at 0: 1048.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 471

Problem: Solve the differential equation dy/dx = 6x with initial condition y(0) = 7.

Direct Answer: $y = (6/2)x^2 + 7$

Detailed Work: Integrate both sides with respect to x: y = integral 6x dx = $(6/2)x^2$ + C. Use y(0)=7: 7=0+C, so C=7.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 472

Problem: Find the area between y=5 and y=x on the interval $0 \le x \le 5$.

Direct Answer: 12.5

Detailed Work: The top function is y=5 and the bottom function is y=x on [0,5]. Area = integral from 0 to 5 of (5-x) dx = $[5x - x^2/2]\_0^5$ = 25 - 12.5 = 12.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 473

Problem: Evaluate lim as $x \to 0$ of sin(1x)/x.

Direct Answer: 1

Detailed Work: Use the standard limit lim as $u \to 0$ of sin(u)/u = 1. Write sin(1x)/x = 1 × [sin(1x)/(1x)]. The bracket approaches 1, so the limit is 1.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 474

Problem: Let $f(x) = 1x^2 + 2x^1$. Find f'(1).

Direct Answer: f'(1) = 4

Detailed Work: Use the power rule: $d/dx[1x^2] = 2x^1$, and $d/dx[2x^1] = 2x^0$. Thus $f'(x) = 2x^1 + 2x^0$. Substitute x=1: f'(1) = 4.

Memory Key: Power rule: $d/dx(x^n) = n x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 475

Problem: Let $f(x)=e^x + 1x^2$. Find f'(1) exactly.

Direct Answer: $e^1 + 2$

Detailed Work: The derivative of e^x is e^x. The derivative of $1x^2$ is 2x. Therefore $f'(x)=e^x+2x$, so $f'(1)=e^1+2$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 476

Problem: A particle has position $s(t) = 1t^3 - 2t^2 + 3t$. Find velocity and acceleration at t=1.

Direct Answer: v(1) = 2, a(1) = 2

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 3t^2 - 4t + 3$. Acceleration is v'(t) = s''(t) = 6t - 4. Substituting t=1 gives v=2 and a=2.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 477

Problem: Find the absolute minimum value of $f(x) = 1(x - 1)^2 + 6$.

Direct Answer: Minimum value = 6 at x = 1

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=1>0, so it opens upward. The vertex is (1,6), and the minimum value is the y-value 6.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 478

Problem: Evaluate the definite integral from 0 to 1 of ($1x^1$ + 1) dx.

Direct Answer: 1.5

Detailed Work: Antiderivative: integral of $1x^1$ is 1/(2) x^2, and integral of 1 is 1x. Evaluate at 1 and subtract the value at 0: 1.5.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 479

Problem: A quantity satisfies dy/dt = 1y and y(0) = 6. Write y(t).

Direct Answer: y(t) = $6e^1$t

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=6, C=6. Thus y(t)=$6e^1$t.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 480

Problem: Find the area between y=2 and y=x on the interval 0 ≤ x ≤ 2.

Direct Answer: 2

Detailed Work: The top function is y=2 and the bottom function is y=x on [0,2]. Area = integral from 0 to 2 of (2-x) dx = [2x - x^2/2]_0^2 = 4 - 2 = 2.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 481

Problem: Evaluate the limit: lim as x → 9 of (x^2 - 81)/(x - 9).

Direct Answer: 18

Detailed Work: Factor the numerator: x^2 - 81 = (x - 9)(x + 9). For x ≠ 9, cancel (x - 9), leaving x + 9. Substitute x = 9: 9 + 9 = 18.

Memory Key: For removable discontinuities, factor first, cancel the common factor, then substitute.

Unit 2: Differentiation - Definition and Basic Rules

Question 482

Problem: Let f(x) = $2x^3$ + $3x^2$. Find f'(2).

Direct Answer: f'(2) = 36

Detailed Work: Use the power rule: d/dx[$2x^3$] = $6x^2$, and d/dx[$3x^2$] = $6x^1$. Thus f'(x) = $6x^2$ + $6x^1$. Substitute x=2: f'(2) = 36.

Memory Key: Power rule: d/dx(x^n) = n x^{n-1}.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 483

Problem: If f(x)=$(3x+2)^3$, find f'(2).

Direct Answer: f'(2) = 576

Detailed Work: Use the chain rule. f'(x) = 3$(3x+2)^2$ × 3 = 9$(3x+2)^2$. At x=2, the inside is 8, so f'(2) = 9$(8)^2$ = 576.

Memory Key: Chain rule: derivative of outside times derivative of inside.

Unit 4: Contextual Applications of Differentiation

Question 484

Problem: A particle has position s(t) = $2t^3$ - $3t^2$ + 4t. Find velocity and acceleration at t=2.

Direct Answer: v(2) = 16, a(2) = 18

Detailed Work: Velocity is the derivative of position: v(t) = s'(t) = $6t^2$ - 6t + 4. Acceleration is v'(t) = s''(t) = 12t - 6. Substituting t=2 gives v=16 and a=18.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 485

Problem: Find the absolute minimum value of f(x) = 2$(x - 2)^2$ + 7.

Direct Answer: Minimum value = 7 at x = 2

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=2>0, so it opens upward. The vertex is (2,7), and the minimum value is the y-value 7.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 486

Problem: Evaluate the definite integral from 0 to 2 of $(2x^2 + 2)$ dx.

Direct Answer: 9.333

Detailed Work: Antiderivative: integral of $2x^2$ is $2/(3)\ x^3$, and integral of 2 is 2x. Evaluate at 2 and subtract the value at 0: 9.333.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 487

Problem: Solve the differential equation dy/dx = 2x with initial condition y(0) = 9.

Direct Answer: $y = (2/2)x^2 + 9$

Detailed Work: Integrate both sides with respect to x: y = integral 2x dx = $(2/2)x^2$ + C. Use y(0)=9: 9=0+C, so C=9.

Memory Key: For dy/dx = g(x), integrate g(x) and use the initial condition.

Unit 8: Applications of Integration

Question 488

Problem: Find the area between y=3 and y=x on the interval $0 \le x \le 3$.

Direct Answer: 4.5

Detailed Work: The top function is y=3 and the bottom function is y=x on [0,3]. Area = integral from 0 to 3 of (3-x) dx = $[3x - x^2/2]\_0^3$ = 9 - 4.5 = 4.5.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 489

Problem: Find c so that f(x) is continuous at x = 7: $f(x)=x^2+c$ for x<7, and f(x)=8x+3 for x≥7.

Direct Answer: c = 10

Detailed Work: For continuity, left value must equal right value at x=7. Left: 7^2 + c = 49 + c. Right: 8(7) + 3 = 59. Set 49+c = 59, so c = 59 - 49 = 10.

Memory Key: Continuity means left-hand limit, right-hand limit, and function value agree.

Unit 2: Differentiation - Definition and Basic Rules

Question 490

Problem: Let $f(x) = 3x^4 + 4x^3$. Find f'(3).

Direct Answer: f'(3) = 432

Detailed Work: Use the power rule: $d/dx[3x^4] = 12x^3$, and $d/dx[4x^3] = 12x^2$. Thus $f'(x) = 12x^3 + 12x^2$. Substitute x=3: f'(3) = 432.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 491

Problem: For the curve $x^2 + y^2 = 25$, find dy/dx at the point (3, 4).

Direct Answer: dy/dx = -3/4 = -0.75

Detailed Work: Differentiate implicitly: 2x + 2y(dy/dx) = 0. Solve for dy/dx: dy/dx = -x/y. At (3,4), dy/dx = -3/4.

Memory Key: Implicit differentiation: every derivative of y includes dy/dx.

Unit 4: Contextual Applications of Differentiation

Question 492

Problem: A particle has position $s(t) = 3t^3 - 4t^2 + 5t$. Find velocity and acceleration at t=3.

Direct Answer: v(3) = 62, a(3) = 46

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 9t^2 - 8t + 5$. Acceleration is v'(t) = s''(t) = 18t - 8. Substituting t=3 gives v=62 and a=46.

Memory Key: Position → velocity → acceleration by differentiating.

Unit 5: Analytical Applications of Differentiation

Question 493

Problem: Find the absolute minimum value of $f(x) = 3(x - 3)^2 + 8$.

Direct Answer: Minimum value = 8 at x = 3

Detailed Work: The function is in vertex form $a(x-h)^2+k$ with a=3>0, so it opens upward. The vertex is (3,8), and the minimum value is the y-value 8.

Memory Key: For a positive quadratic in vertex form, the vertex gives the minimum.

Unit 6: Integration and Accumulation of Change

Question 494

Problem: Evaluate the definite integral from 0 to 3 of $(3x^3 + 3)$ dx.

Direct Answer: 69.75

Detailed Work: Antiderivative: integral of $3x^3$ is $3/(4)\ x^4$, and integral of 3 is 3x. Evaluate at 3 and subtract the value at 0: 69.75.

Memory Key: Definite integrals use antiderivative at upper limit minus lower limit.

Unit 7: Differential Equations

Question 495

Problem: A quantity satisfies dy/dt = 3y and y(0) = 8. Write y(t).

Direct Answer: $y(t) = 8e^3t$

Detailed Work: The equation dy/dt = ky has exponential solution y = Cekt. Since y(0)=8, C=8. Thus $y(t)=8e^3t$.

Memory Key: dy/dt = ky gives exponential growth/decay.

Unit 8: Applications of Integration

Question 496

Problem: Find the area between y=4 and y=x on the interval $0 \le x \le 4$.

Direct Answer: 8

Detailed Work: The top function is y=4 and the bottom function is y=x on [0,4]. Area = integral from 0 to 4 of (4-x) dx = $[4x - x^2/2]\_0^4 = 16 - 8 = 8$.

Memory Key: Area between curves = integral of top minus bottom.

Unit 1: Limits and Continuity

Question 497

Problem: Evaluate lim as x → 0 of sin(4x)/x.

Direct Answer: 4

Detailed Work: Use the standard limit lim as u → 0 of sin(u)/u = 1. Write sin(4x)/x = 4 × [sin(4x)/(4x)]. The bracket approaches 1, so the limit is 4.

Memory Key: Convert trig limits to the standard form sin(u)/u.

Unit 2: Differentiation - Definition and Basic Rules

Question 498

Problem: Let $f(x) = 4x^5 + 5x^1$. Find f'(4).

Direct Answer: f'(4) = 5125

Detailed Work: Use the power rule: $d/dx[4x^5] = 20x^4$, and $d/dx[5x^1] = 5x^0$. Thus $f'(x) = 20x^4 + 5x^0$. Substitute x=4: f'(4) = 5125.

Memory Key: Power rule: $d/dx(x^n) = n\ x^{n-1}$.

Unit 3: Differentiation - Composite, Implicit, and Inverse Functions

Question 499

Problem: Let $f(x)=e^x + 4x^2$. Find f'(4) exactly.

Direct Answer: $e^4 + 32$

Detailed Work: The derivative of e^x is e^x. The derivative of $4x^2$ is 8x. Therefore $f'(x)=e^x+8x$, so $f'(4)=e^4+32$.

Memory Key: Know core derivative rules: e^x stays e^x, and x^2 becomes 2x.

Unit 4: Contextual Applications of Differentiation

Question 500

Problem: A particle has position $s(t) = 4t^3 - 5t^2 + 6t$. Find velocity and acceleration at $t=4$.

Direct Answer: $v(4) = 158$, $a(4) = 86$

Detailed Work: Velocity is the derivative of position: $v(t) = s'(t) = 12t^2 - 10t + 6$. Acceleration is $v'(t) = s''(t) = 24t - 10$. Substituting $t=4$ gives $v=158$ and $a=86$.

Memory Key: Position → velocity → acceleration by differentiating.

AP Physics 1

Unit 1: Kinematics

Question 1

Problem: A cart starts with velocity 2 m/s and accelerates at 2 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 8 m/s, displacement = 15 m

Detailed Work: Use constant-acceleration equations. Final velocity: $v = v0 + at = 2+2(3) = 8$ m/s. Displacement: delta $x = v0t + (½)at^2 = 2(3) + 0.5(2)(3)^2 = 15$ m.

Memory Key: For constant acceleration, use v=v0+at and delta $x=v0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 2

Problem: A 3 kg object accelerates at 2 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 6 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=3 kg and a=2 m/s^2: F_net = 3(2) = 6 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 3

Problem: Find the kinetic energy of a 2 kg object moving at 3 m/s.

Direct Answer: 9 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(2)(3)^2 = 9$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 4

Problem: Find the momentum of a 2 kg object moving at 3 m/s.

Direct Answer: 6 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=3 m/s: p=2(3)=6 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 5

Problem: A force of 11 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 2.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(11) = 2.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 6

Problem: A rotating object has rotational inertia 2 $kg*m^2$ and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 9 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = $0.5(2)(3)^2 = 9$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 7

Problem: A mass-spring oscillator has mass 2 kg and spring constant 25 N/m. Find the period.

Direct Answer: 1.777 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=25: T = 2*π*√(2/25) = 1.777 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 8

Problem: Find the gauge pressure at a depth of 2 m in water. Use rho = 1000 kg/m³.

Direct Answer: 19600 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(2) = 19600 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 9

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: h = 0.5gt², so t = √(2h/g) = √(2(7)/9.8) = 1.195 s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 10

Problem: A 7 kg box slides on a level surface with kinetic friction coefficient 0.25. Find the friction force magnitude.

Direct Answer: 17.15 N

Detailed Work: On a level surface, normal force N = mg = 7(9.8). Kinetic friction is f_k = mu_k N = 0.25(7)(9.8) = 17.15 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 11

Problem: A 4 kg object is lifted 5 m. Find the gain in gravitational potential energy.

Direct Answer: 196 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 4(9.8)(5) = 196 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 12

Problem: A 3 kg cart moving at 5 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 2.143 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(5) + 4(0) = 15. Total mass = 7. Final speed v_f = 15/7 = 2.143 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 13

Problem: A net torque of 4 N*m acts on an object with rotational inertia 3 kg*m². Find the angular acceleration.

Direct Answer: 1.333 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 4/3 = 1.333 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 14

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m² and angular speed 3 rad/s.

Direct Answer: 12 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(3)=12 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 15

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=1.5 m and $g=9.8\ m/s^2$: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 16

Problem: An object displaces $0.003\ m^3$ of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 17

Problem: A cart starts with velocity 4 m/s and accelerates at $4\ m/s^2$ for 5 s. Find its final velocity and displacement.

Direct Answer: v = 24 m/s, displacement = 70 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 4+4(5) = 24 m/s. Displacement: delta x = v0t + $(½)at^2$ = 4(5) + $0.5(4)(5)^2$ = 70 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=$v0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 18

Problem: A force of 26 N pulls a 6 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: $3.753\ m/s^2$

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 26(0.866). Then a = F_x/m = 22.516/6 = $3.753\ m/s^2$.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 19

Problem: A constant horizontal force of 13 N moves a box 5 m in the same direction. Find the work done.

Direct Answer: 65 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=13(5)=65 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 20

Problem: Find the momentum of a 4 kg object moving at 5 m/s.

Direct Answer: 20 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=5 m/s: p=4(5)=20 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 21

Problem: A force of 13 N is applied perpendicular to a wrench 0.4 m from the πvot. Find the torque magnitude.

Direct Answer: 5.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.4(13) = 5.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 22

Problem: A rotating object has rotational inertia $4\ kg*m^2$ and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 50 J

Detailed Work: Rotational kinetic energy is K_rot = $(½)I\ omega^2$. Substitute: K_rot = $0.5(4)(5)^2$ = 50 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 23

Problem: A mass-spring oscillator has mass 4 kg and spring constant 35 N/m. Find the period.

Direct Answer: 2.124 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=35: T = 2*π*√(4/35) = 2.124 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 24

Problem: An ideal incompressible fluid flows through a πpe. If A1=5 cm^2, v1=4 m/s, and A2=4 cm^2, find v2.

Direct Answer: 5 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 5(4)/4 = 5 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 25

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(9)/9.8) = 1.355 s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 26

Problem: A 6 kg object accelerates at 5 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 30 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=6 kg and a=5 m/s^2: F_net = 6(5) = 30 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 27

Problem: Find the kinetic energy of a 5 kg object moving at 6 m/s.

Direct Answer: 90 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = $0.5(5)(6)^2$ = 90 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 28

Problem: A 5 kg cart moving at 7 m/s sticks to a 6 kg cart at rest. Find their final speed.

Direct Answer: 3.182 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 5(7) + 6(0) = 35. Total mass = 11. Final speed v_f = 35/11 = 3.182 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 29

Problem: A net torque of 6 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 1.2 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 6/5 = 1.2 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 30

Problem: Find the angular momentum of a rigid body with rotational inertia 6 kg*m² and angular speed 5 rad/s.

Direct Answer: 30 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(5)=30 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 31

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=2.5 m and g=9.8 m/s²: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 32

Problem: Find the gauge pressure at a depth of 5 m in water. Use rho = 1000 kg/m³.

Direct Answer: 49000 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(5) = 49000 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 33

Problem: A cart starts with velocity 6 m/s and accelerates at 1 m/s² for 7 s. Find its final velocity and displacement.

Direct Answer: v = 13 m/s, displacement = 66.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+1(7) = 13 m/s. Displacement: delta x = v0t + (½)at² = 6(7) + 0.5(1)(7)² = 66.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 34

Problem: A 10 kg box slides on a level surface with kinetic friction coefficient 0.1. Find the friction force magnitude.

Direct Answer: 9.8 N

Detailed Work: On a level surface, normal force N = mg = 10(9.8). Kinetic friction is f_k = mu_k N = 0.1(10)(9.8) = 9.8 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 35

Problem: A 2 kg object is lifted 8 m. Find the gain in gravitational potential energy.

Direct Answer: 156.8 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 2(9.8)(8) = 156.8 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 36

Problem: Find the momentum of a 6 kg object moving at 7 m/s.

Direct Answer: 42 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=7 m/s: p=6(7)=42 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 37

Problem: A force of 15 N is applied perpendicular to a wrench 0.1 m from the πvot. Find the torque magnitude.

Direct Answer: 1.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.1(15) = 1.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 38

Problem: A rotating object has rotational inertia 6 kg*m^2 and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 147 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = $0.5(6)(7)^2$ = 147 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 39

Problem: A mass-spring oscillator has mass 6 kg and spring constant 45 N/m. Find the period.

Direct Answer: 2.294 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=45: T = 2*π*√(6/45) = 2.294 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 40

Problem: An object displaces 0.006 m^3 of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 41

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(11)/9.8) = 1.498 s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 42

Problem: A force of 32 N pulls a 9 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 3.079 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 32(0.866). Then a = F_x/m = 27.712/9 = 3.079 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 43

Problem: A constant horizontal force of 16 N moves a box 8 m in the same direction. Find the work done.

Direct Answer: 128 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=16(8)=128 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 44

Problem: A 2 kg cart moving at 9 m/s sticks to a 3 kg cart at rest. Find their final speed.

Direct Answer: 3.6 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 2(9) + 3(0) = 18. Total mass = 5. Final speed v_f = 18/5 = 3.6 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 45

Problem: A net torque of 8 N*m acts on an object with rotational inertia 1 kg*m². Find the angular acceleration.

Direct Answer: 8 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 8/1 = 8 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 46

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m² and angular speed 7 rad/s.

Direct Answer: 56 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(7)=56 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 47

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=3.5 m and g=9.8 m/s²: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 48

Problem: An ideal incompressible fluid flows through a πpe. If A1=3 cm², v1=7 m/s, and A2=3 cm², find v2.

Direct Answer: 7 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 3(7)/3 = 7 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 49

Problem: A cart starts with velocity 8 m/s and accelerates at 3 m/s² for 3 s. Find its final velocity and displacement.

Direct Answer: v = 17 m/s, displacement = 37.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+3(3) = 17 m/s. Displacement: delta x = v0t + (½)at² = 8(3) + 0.5(3)(3)² = 37.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 50

Problem: A 9 kg object accelerates at 3 m/s² on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 27 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=9 kg and a=3 m/s²: F_net = 9(3) = 27 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 51

Problem: Find the kinetic energy of a 2 kg object moving at 9 m/s.

Direct Answer: 81 J

Detailed Work: Kinetic energy is K = (½)mv². Substitute: K = 0.5(2)(9)² = 81 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 52

Problem: Find the momentum of a 8 kg object moving at 9 m/s.

Direct Answer: 72 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=9 m/s: p=8(9)=72 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 53

Problem: A force of 17 N is applied perpendicular to a wrench 0.3 m from the πvot. Find the torque magnitude.

Direct Answer: 5.1 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.3(17) = 5.1 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 54

Problem: A rotating object has rotational inertia 1 kg*m^2 and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 40.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = $0.5(1)(9)^2$ = 40.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 55

Problem: A mass-spring oscillator has mass 2 kg and spring constant 55 N/m. Find the period.

Direct Answer: 1.198 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=55: T = 2*π*√(2/55) = 1.198 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 56

Problem: Find the gauge pressure at a depth of 8 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 78400 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(8) = 78400 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 57

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(5)/9.8) = 1.01 s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 58

Problem: A 5 kg box slides on a level surface with kinetic friction coefficient 0.3. Find the friction force magnitude.

Direct Answer: 14.7 N

Detailed Work: On a level surface, normal force N = mg = 5(9.8). Kinetic friction is f_k = mu_k N = 0.3(5)(9.8) = 14.7 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 59

Problem: A 5 kg object is lifted 3 m. Find the gain in gravitational potential energy.

Direct Answer: 147 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 5(9.8)(3) = 147 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 60

Problem: A 4 kg cart moving at 4 m/s sticks to a 5 kg cart at rest. Find their final speed.

Direct Answer: 1.778 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 4(4) + 5(0) = 16. Total mass = 9. Final speed v_f = 16/9 = 1.778 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 61

Problem: A net torque of 10 N*m acts on an object with rotational inertia 3 kg*m^2. Find the angular acceleration.

Direct Answer: 3.333 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 10/3 = 3.333 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 62

Problem: Find the angular momentum of a rigid body with rotational inertia 2 kg*m^2 and angular speed 9 rad/s.

Direct Answer: 18 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(9)=18 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 63

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=0.5 m and g=9.8 m/s^2: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 64

Problem: An object displaces 0.009 m^3 of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.009) = 88.2 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 65

Problem: A cart starts with velocity 2 m/s and accelerates at 5 m/s^2 for 5 s. Find its final velocity and displacement.

Direct Answer: v = 27 m/s, displacement = 72.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 2+5(5) = 27 m/s. Displacement: delta x = v0t + (½)at^2 = 2(5) + 0.5(5)$(5)^2$ = 72.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 66

Problem: A force of 38 N pulls a 4 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 8.227 m/s^2

Detailed Work: Only the horizontal component accelerates the object: $F_x = F \cos 30 = 38(0.866)$. Then $a = F_x/m = 32.908/4 = 8.227$ m/s².

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 67

Problem: A constant horizontal force of 19 N moves a box 3 m in the same direction. Find the work done.

Direct Answer: 57 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=19(3)=57 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 68

Problem: Find the momentum of a 2 kg object moving at 11 m/s.

Direct Answer: 22 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=11 m/s: p=2(11)=22 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 69

Problem: A force of 19 N is applied perpendicular to a wrench 0.5 m from the πvot. Find the torque magnitude.

Direct Answer: 9.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.5(19) = 9.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 70

Problem: A rotating object has rotational inertia 3 kg*m² and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 13.5 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(3)(3)^2 = 13.5$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 71

Problem: A mass-spring oscillator has mass 4 kg and spring constant 65 N/m. Find the period.

Direct Answer: 1.559 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=65: T = 2*π*√(4/65) = 1.559 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 72

Problem: An ideal incompressible fluid flows through a πpe. If A1=6 cm², v1=2 m/s, and A2=2 cm², find v2.

Direct Answer: 6 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 6(2)/2 = 6 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 73

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so t = √(2h/g) = √(2(7)/9.8) = 1.195 s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 74

Problem: A 3 kg object accelerates at 1 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 3 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=3 kg and a=1 m/s^2: F_net = 3(1) = 3 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 75

Problem: Find the kinetic energy of a 5 kg object moving at 2 m/s.

Direct Answer: 10 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = $0.5(5)(2)^2$ = 10 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 76

Problem: A 1 kg cart moving at 6 m/s sticks to a 2 kg cart at rest. Find their final speed.

Direct Answer: 2 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 1(6) + 2(0) = 6. Total mass = 3. Final speed v_f = 6/3 = 2 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 77

Problem: A net torque of 2 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 0.4 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 2/5 = 0.4 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 78

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m^2 and angular speed 2 rad/s.

Direct Answer: 8 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(2)=8 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 79

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=1.5 m and g=9.8 m/s^2: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 80

Problem: Find the gauge pressure at a depth of 1 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 9800 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(1) = 9800 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 81

Problem: A cart starts with velocity 4 m/s and accelerates at 2 m/s^2 for 7 s. Find its final velocity and displacement.

Direct Answer: v = 18 m/s, displacement = 77 m

Detailed Work: Use constant-acceleration equations. Final velocity: $v = v0 + at = 4+2(7) = 18$ m/s. Displacement: delta $x = v0t + (½)at^2 = 4(7) + 0.5(2)(7)^2 = 77$ m.

Memory Key: For constant acceleration, use $v=v0+at$ and delta $x=v0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 82

Problem: A 8 kg box slides on a level surface with kinetic friction coefficient 0.2. Find the friction force magnitude.

Direct Answer: 15.68 N

Detailed Work: On a level surface, normal force N = mg = 8(9.8). Kinetic friction is $f_k = mu_k N = 0.2(8)(9.8) = 15.68$ N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 83

Problem: A 3 kg object is lifted 6 m. Find the gain in gravitational potential energy.

Direct Answer: 176.4 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 3(9.8)(6) = 176.4 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 84

Problem: Find the momentum of a 4 kg object moving at 13 m/s.

Direct Answer: 52 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=13 m/s: p=4(13)=52 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 85

Problem: A force of 21 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 4.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(21) = 4.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 86

Problem: A rotating object has rotational inertia 5 $kg*m^2$ and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 62.5 J

Detailed Work: Rotational kinetic energy is $K_rot = (½)I\ omega^2$. Substitute: $K_rot = 0.5(5)(5)^2 = 62.5$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 87

Problem: A mass-spring oscillator has mass 6 kg and spring constant 25 N/m. Find the period.

Direct Answer: 3.078 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=25: T = 2*π*√(6/25) = 3.078 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 88

Problem: An object displaces 0.003 m^3 of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 89

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(9)/9.8} = 1.355$ s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 90

Problem: A force of 44 N pulls a 7 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 5.444 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 44(0.866). Then a = F_x/m = 38.104/7 = 5.444 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 91

Problem: A constant horizontal force of 22 N moves a box 6 m in the same direction. Find the work done.

Direct Answer: 132 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=22(6)=132 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 92

Problem: A 3 kg cart moving at 8 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 3.429 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(8) + 4(0) = 24. Total mass = 7. Final speed v_f = 24/7 = 3.429 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 93

Problem: A net torque of 4 N*m acts on an object with rotational inertia 1 $kg*m^2$. Find the angular acceleration.

Direct Answer: 4 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 4/1 = 4 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 94

Problem: Find the angular momentum of a rigid body with rotational inertia 6 $kg*m^2$ and angular speed 4 rad/s.

Direct Answer: 24 $kg*m^2/s$

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(4)=24 $kg*m^2/s$.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 95

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{L/g}$. Substitute L=2.5 m and g=9.8 m/s^2: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 96

Problem: An ideal incompressible fluid flows through a πpe. If A1=4 cm², v1=5 m/s, and A2=1 cm², find v2.

Direct Answer: 20 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 4(5)/1 = 20 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 97

Problem: A cart starts with velocity 6 m/s and accelerates at 4 m/s² for 3 s. Find its final velocity and displacement.

Direct Answer: v = 18 m/s, displacement = 36 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+4(3) = 18 m/s. Displacement: delta x = v0t + (½)at² = 6(3) + 0.5(4)(3)² = 36 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 98

Problem: A 6 kg object accelerates at 4 m/s² on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 24 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=6 kg and a=4 m/s²: F_net = 6(4) = 24 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 99

Problem: Find the kinetic energy of a 2 kg object moving at 5 m/s.

Direct Answer: 25 J

Detailed Work: Kinetic energy is K = (½)mv². Substitute: K = 0.5(2)(5)² = 25 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 100

Problem: Find the momentum of a 6 kg object moving at 3 m/s.

Direct Answer: 18 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=3 m/s: p=6(3)=18 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 101

Problem: A force of 23 N is applied perpendicular to a wrench 0.4 m from the πvot. Find the torque magnitude.

Direct Answer: 9.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.4(23) = 9.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 102

Problem: A rotating object has rotational inertia 7 kg*m² and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 171.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I omega². Substitute: K_rot = 0.5(7)(7)² = 171.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 103

Problem: A mass-spring oscillator has mass 2 kg and spring constant 35 N/m. Find the period.

Direct Answer: 1.502 s

Detailed Work: For a mass-spring oscillator, $T = 2*\pi*\sqrt{(m/k)}$. Substitute m=2, k=35: $T = 2*\pi*\sqrt{(2/35)} = 1.502$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 104

Problem: Find the gauge pressure at a depth of 4 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 39200 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(4) = 39200 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 105

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{(2h/g)} = \sqrt{(2(11)/9.8)} = 1.498$ s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 106

Problem: A 11 kg box slides on a level surface with kinetic friction coefficient 0.4. Find the friction force magnitude.

Direct Answer: 43.12 N

Detailed Work: On a level surface, normal force N = mg = 11(9.8). Kinetic friction is f_k = mu_k N = 0.4(11)(9.8) = 43.12 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 107

Problem: A 6 kg object is lifted 9 m. Find the gain in gravitational potential energy.

Direct Answer: 529.2 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 6(9.8)(9) = 529.2 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 108

Problem: A 5 kg cart moving at 3 m/s sticks to a 6 kg cart at rest. Find their final speed.

Direct Answer: 1.364 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 5(3) + 6(0) = 15. Total mass = 11. Final speed v_f = 15/11 = 1.364 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 109

Problem: A net torque of 6 N*m acts on an object with rotational inertia 3 kg*m^2. Find the angular acceleration.

Direct Answer: 2 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 6/3 = 2 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 110

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m^2 and angular speed 6 rad/s.

Direct Answer: 48 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(6)=48 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 111

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=3.5 m and g=9.8 m/s²: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 112

Problem: An object displaces 0.006 m³ of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 113

Problem: A cart starts with velocity 8 m/s and accelerates at 1 m/s² for 5 s. Find its final velocity and displacement.

Direct Answer: v = 13 m/s, displacement = 52.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+1(5) = 13 m/s. Displacement: delta x = v0t + (½)at² = 8(5) + 0.5(1)(5)² = 52.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 114

Problem: A force of 50 N pulls a 10 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 4.33 m/s²

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 50(0.866). Then a = F_x/m = 43.3/10 = 4.33 m/s².

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 115

Problem: A constant horizontal force of 25 N moves a box 9 m in the same direction. Find the work done.

Direct Answer: 225 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=25(9)=225 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 116

Problem: Find the momentum of a 8 kg object moving at 5 m/s.

Direct Answer: 40 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=5 m/s: p=8(5)=40 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 117

Problem: A force of 25 N is applied perpendicular to a wrench 0.1 m from the πvot. Find the torque magnitude.

Direct Answer: 2.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.1(25) = 2.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 118

Problem: A rotating object has rotational inertia 2 kg*m^2 and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 81 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(2)$(9)^2$ = 81 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 119

Problem: A mass-spring oscillator has mass 4 kg and spring constant 45 N/m. Find the period.

Direct Answer: 1.873 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=45: T = 2*π*√(4/45) = 1.873 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 120

Problem: An ideal incompressible fluid flows through a πpe. If A1=2 cm^2, v1=8 m/s, and A2=4 cm^2, find v2.

Direct Answer: 4 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 2(8)/4 = 4 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 121

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(5)/9.8) = 1.01 s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 122

Problem: A 9 kg object accelerates at 2 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 18 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=9 kg and a=2 m/s^2: F_net = 9(2) = 18 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 123

Problem: Find the kinetic energy of a 5 kg object moving at 8 m/s.

Direct Answer: 160 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = 0.5(5)$(8)^2$ = 160 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 124

Problem: A 2 kg cart moving at 5 m/s sticks to a 3 kg cart at rest. Find their final speed.

Direct Answer: 2 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 2(5) + 3(0) = 10. Total mass = 5. Final speed v_f = 10/5 = 2 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 125

Problem: A net torque of 8 N*m acts on an object with rotational inertia 5 kg*m². Find the angular acceleration.

Direct Answer: 1.6 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 8/5 = 1.6 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 126

Problem: Find the angular momentum of a rigid body with rotational inertia 2 kg*m² and angular speed 8 rad/s.

Direct Answer: 16 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(8)=16 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 127

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=0.5 m and g=9.8 m/s²: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 128

Problem: Find the gauge pressure at a depth of 7 m in water. Use rho = 1000 kg/m³.

Direct Answer: 68600 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(7) = 68600 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 129

Problem: A cart starts with velocity 2 m/s and accelerates at 3 m/s² for 7 s. Find its final velocity and displacement.

Direct Answer: v = 23 m/s, displacement = 87.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 2+3(7) = 23 m/s. Displacement: delta x = v0t + (½)at² = 2(7) + 0.5(3)(7)² = 87.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 130

Problem: A 6 kg box slides on a level surface with kinetic friction coefficient 0.25. Find the friction force magnitude.

Direct Answer: 14.7 N

Detailed Work: On a level surface, normal force N = mg = 6(9.8). Kinetic friction is f_k = mu_k N = 0.25(6)(9.8) = 14.7 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 131

Problem: A 4 kg object is lifted 4 m. Find the gain in gravitational potential energy.

Direct Answer: 156.8 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 4(9.8)(4) = 156.8 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 132

Problem: Find the momentum of a 2 kg object moving at 7 m/s.

Direct Answer: 14 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=7 m/s: p=2(7)=14 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 133

Problem: A force of 27 N is applied perpendicular to a wrench 0.3 m from the πvot. Find the torque magnitude.

Direct Answer: 8.1 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.3(27) = 8.1 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 134

Problem: A rotating object has rotational inertia 4 kg*m^2 and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 18 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I omega^2. Substitute: K_rot = 0.5(4)(3)^2 = 18 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 135

Problem: A mass-spring oscillator has mass 6 kg and spring constant 55 N/m. Find the period.

Direct Answer: 2.075 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=55: T = 2*π*√(6/55) = 2.075 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 136

Problem: An object displaces 0.009 m^3 of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.009) = 88.2 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 137

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(7)/9.8) = 1.195 s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 138

Problem: A force of 56 N pulls a 5 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 9.699 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 56(0.866). Then a = F_x/m = 48.496/5 = 9.699 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 139

Problem: A constant horizontal force of 28 N moves a box 4 m in the same direction. Find the work done.

Direct Answer: 112 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=28(4)=112 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 140

Problem: A 4 kg cart moving at 7 m/s sticks to a 5 kg cart at rest. Find their final speed.

Direct Answer: 3.111 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 4(7) + 5(0) = 28. Total mass = 9. Final speed v_f = 28/9 = 3.111 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 141

Problem: A net torque of 10 N*m acts on an object with rotational inertia 1 kg*m^2. Find the angular acceleration.

Direct Answer: 10 rad/s^2

Detailed Work: Rotational Newton's second law is τ_{net} = I alpha. Solve alpha = tau/I = 10/1 = 10 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 142

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m^2 and angular speed 1 rad/s.

Direct Answer: 4 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(1)=4 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 143

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=1.5 m and g=9.8 m/s^2: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 144

Problem: An ideal incompressible fluid flows through a πpe. If A1=5 cm^2, v1=3 m/s, and A2=3 cm^2, find v2.

Direct Answer: 5 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 5(3)/3 = 5 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 145

Problem: A cart starts with velocity 4 m/s and accelerates at 5 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 19 m/s, displacement = 34.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 4+5(3) = 19 m/s. Displacement: delta x = v0t + (½)at^2 = 4(3) + 0.5(5)(3)^2 = 34.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 146

Problem: A 3 kg object accelerates at 5 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 15 N

Detailed Work: Newton's second law gives F_{net} = ma. Substitute m=3 kg and a=5 m/s^2: F_{net} = 3(5) = 15 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 147

Problem: Find the kinetic energy of a 2 kg object moving at 11 m/s.

Direct Answer: 121 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(2)(11)^2 = 121$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 148

Problem: Find the momentum of a 4 kg object moving at 9 m/s.

Direct Answer: 36 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=9 m/s: p=4(9)=36 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 149

Problem: A force of 29 N is applied perpendicular to a wrench 0.5 m from the πvot. Find the torque magnitude.

Direct Answer: 14.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.5(29) = 14.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 150

Problem: A rotating object has rotational inertia 6 $kg*m^2$ and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 75 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(6)(5)^2 = 75$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 151

Problem: A mass-spring oscillator has mass 2 kg and spring constant 65 N/m. Find the period.

Direct Answer: 1.102 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=65: T = 2*π*√(2/65) = 1.102 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 152

Problem: Find the gauge pressure at a depth of 10 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 98000 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(10) = 98000 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 153

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so t = √(2h/g) = √(2(9)/9.8) = 1.355 s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 154

Problem: A 9 kg box slides on a level surface with kinetic friction coefficient 0.1. Find the friction force magnitude.

Direct Answer: 8.82 N

Detailed Work: On a level surface, normal force N = mg = 9(9.8). Kinetic friction is $f_k = \mu_k N$ = 0.1(9)(9.8) = 8.82 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 155

Problem: A 2 kg object is lifted 7 m. Find the gain in gravitational potential energy.

Direct Answer: 137.2 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 2(9.8)(7) = 137.2 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 156

Problem: A 1 kg cart moving at 9 m/s sticks to a 2 kg cart at rest. Find their final speed.

Direct Answer: 3 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 1(9) + 2(0) = 9. Total mass = 3. Final speed v_f = 9/3 = 3 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 157

Problem: A net torque of 2 N*m acts on an object with rotational inertia 3 kg*m^2. Find the angular acceleration.

Direct Answer: 0.667 rad/s^2

Detailed Work: Rotational Newton's second law is tau_{net} = I alpha. Solve alpha = tau/I = 2/3 = 0.667 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 158

Problem: Find the angular momentum of a rigid body with rotational inertia 6 kg*m^2 and angular speed 3 rad/s.

Direct Answer: 18 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(3)=18 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 159

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{L/g}$. Substitute L=2.5 m and g=9.8 m/s^2: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 160

Problem: An object displaces 0.003 m^3 of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 161

Problem: A cart starts with velocity 6 m/s and accelerates at 2 m/s^2 for 5 s. Find its final velocity and displacement.

Direct Answer: v = 16 m/s, displacement = 55 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+2(5) = 16 m/s. Displacement: delta x = v0t + (½)at^2 = 6(5) + 0.5(2)$(5)^2$ = 55 m.

Memory Key: For constant acceleration, use $v=v_0+at$ and delta $x=v_0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 162

Problem: A force of 22 N pulls a 8 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 2.382 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 22(0.866). Then a = F_x/m = 19.052/8 = 2.382 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 163

Problem: A constant horizontal force of 11 N moves a box 7 m in the same direction. Find the work done.

Direct Answer: 77 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=11(7)=77 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 164

Problem: Find the momentum of a 6 kg object moving at 11 m/s.

Direct Answer: 66 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=11 m/s: p=6(11)=66 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 165

Problem: A force of 31 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 6.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(31) = 6.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 166

Problem: A rotating object has rotational inertia 1 kg*m^2 and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 24.5 J

Detailed Work: Rotational kinetic energy is K_{rot} = (½)I $omega^2$. Substitute: K_{rot} = $0.5(1)(7)^2$ = 24.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 167

Problem: A mass-spring oscillator has mass 4 kg and spring constant 25 N/m. Find the period.

Direct Answer: 2.513 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=25: T = 2*π*√(4/25) = 2.513 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 168

Problem: An ideal incompressible fluid flows through a πpe. If A1=3 cm^2, v1=6 m/s, and A2=2 cm^2, find v2.

Direct Answer: 9 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 3(6)/2 = 9 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 169

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(11)/9.8} = 1.498$ s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 170

Problem: A 6 kg object accelerates at 3 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 18 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=6 kg and a=3 m/s^2: F_net = 6(3) = 18 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 171

Problem: Find the kinetic energy of a 5 kg object moving at 4 m/s.

Direct Answer: 40 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(5)(4)^2 = 40$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 172

Problem: A 3 kg cart moving at 4 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 1.714 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(4) + 4(0) = 12. Total mass = 7. Final speed v_f = 12/7 = 1.714 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 173

Problem: A net torque of 4 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 0.8 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 4/5 = 0.8 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 174

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m^2 and angular speed 5 rad/s.

Direct Answer: 40 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(5)=40 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 175

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=3.5 m and g=9.8 m/s^2: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 176

Problem: Find the gauge pressure at a depth of 3 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 29400 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(3) = 29400 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 177

Problem: A cart starts with velocity 8 m/s and accelerates at 4 m/s^2 for 7 s. Find its final velocity and displacement.

Direct Answer: v = 36 m/s, displacement = 154 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+4(7) = 36 m/s. Displacement: delta x = v0t + (½)at^2 = 8(7) + 0.5(4)$(7)^2$ = 154 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 178

Problem: A 12 kg box slides on a level surface with kinetic friction coefficient 0.3. Find the friction force magnitude.

Direct Answer: 35.28 N

Detailed Work: On a level surface, normal force N = mg = 12(9.8). Kinetic friction is f_k = mu_k N = 0.3(12)(9.8) = 35.28 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 179

Problem: A 5 kg object is lifted 10 m. Find the gain in gravitational potential energy.

Direct Answer: 490 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 5(9.8)(10) = 490 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 180

Problem: Find the momentum of a 8 kg object moving at 13 m/s.

Direct Answer: 104 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=13 m/s: p=8(13)=104 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 181

Problem: A force of 33 N is applied perpendicular to a wrench 0.4 m from the πvot. Find the torque magnitude.

Direct Answer: 13.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.4(33) = 13.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 182

Problem: A rotating object has rotational inertia 3 kg*m^2 and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 121.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(3)$(9)^2$ = 121.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 183

Problem: A mass-spring oscillator has mass 6 kg and spring constant 35 N/m. Find the period.

Direct Answer: 2.601 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=35: T = 2*π*√(6/35) = 2.601 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 184

Problem: An object displaces 0.006 m^3 of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 185

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(5)/9.8) = 1.01 s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 186

Problem: A force of 28 N pulls a 3 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 8.083 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 28(0.866). Then a = F_x/m = 24.248/3 = 8.083 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 187

Problem: A constant horizontal force of 14 N moves a box 2 m in the same direction. Find the work done.

Direct Answer: 28 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=14(2)=28 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 188

Problem: A 5 kg cart moving at 6 m/s sticks to a 6 kg cart at rest. Find their final speed.

Direct Answer: 2.727 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 5(6) + 6(0) = 30. Total mass = 11. Final speed v_f = 30/11 = 2.727 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 189

Problem: A net torque of 6 N*m acts on an object with rotational inertia 1 kg*m^2. Find the angular acceleration.

Direct Answer: 6 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 6/1 = 6 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 190

Problem: Find the angular momentum of a rigid body with rotational inertia 2 kg*m^2 and angular speed 7 rad/s.

Direct Answer: 14 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(7)=14 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 191

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=0.5 m and g=9.8 m/s^2: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 192

Problem: An ideal incompressible fluid flows through a πpe. If A1=6 cm^2, v1=1 m/s, and A2=1 cm^2, find v2.

Direct Answer: 6 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 6(1)/1 = 6 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 193

Problem: A cart starts with velocity 2 m/s and accelerates at 1 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 5 m/s, displacement = 10.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 2+1(3) = 5 m/s. Displacement: delta x = v0t + (½)at^2 = 2(3) + 0.5(1)$(3)^2$ = 10.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 194

Problem: A 9 kg object accelerates at 1 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 9 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=9 kg and a=1 m/s^2: F_net = 9(1) = 9 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 195

Problem: Find the kinetic energy of a 2 kg object moving at 7 m/s.

Direct Answer: 49 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = 0.5(2)$(7)^2$ = 49 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 196

Problem: Find the momentum of a 2 kg object moving at 3 m/s.

Direct Answer: 6 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=3 m/s: p=2(3)=6 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 197

Problem: A force of 35 N is applied perpendicular to a wrench 0.1 m from the πvot. Find the torque magnitude.

Direct Answer: 3.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.1(35) = 3.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 198

Problem: A rotating object has rotational inertia 5 kg*m^2 and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 22.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I ω^2. Substitute: K_rot = $0.5(5)(3)^2$ = 22.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 199

Problem: A mass-spring oscillator has mass 2 kg and spring constant 45 N/m. Find the period.

Direct Answer: 1.325 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=45: T = 2*π*√(2/45) = 1.325 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 200

Problem: Find the gauge pressure at a depth of 6 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 58800 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(6) = 58800 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 201

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(7)/9.8) = 1.195 s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 202

Problem: A 7 kg box slides on a level surface with kinetic friction coefficient 0.2. Find the friction force magnitude.

Direct Answer: 13.72 N

Detailed Work: On a level surface, normal force N = mg = 7(9.8). Kinetic friction is f_k = mu_k N = 0.2(7)(9.8) = 13.72 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 203

Problem: A 3 kg object is lifted 5 m. Find the gain in gravitational potential energy.

Direct Answer: 147 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 3(9.8)(5) = 147 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 204

Problem: A 2 kg cart moving at 8 m/s sticks to a 3 kg cart at rest. Find their final speed.

Direct Answer: 3.2 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 2(8) + 3(0) = 16. Total mass = 5. Final speed v_f = 16/5 = 3.2 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 205

Problem: A net torque of 8 N*m acts on an object with rotational inertia 3 $kg*m^2$. Find the angular acceleration.

Direct Answer: 2.667 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 8/3 = 2.667 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 206

Problem: Find the angular momentum of a rigid body with rotational inertia 4 $kg*m^2$ and angular speed 9 rad/s.

Direct Answer: 36 $kg*m^2/s$

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(9)=36 $kg*m^2/s$.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 207

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=1.5 m and g=9.8 m/s^2: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 208

Problem: An object displaces 0.009 m^3 of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.009) = 88.2 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 209

Problem: A cart starts with velocity 4 m/s and accelerates at 3 m/s^2 for 5 s. Find its final velocity and displacement.

Direct Answer: v = 19 m/s, displacement = 57.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 4+3(5) = 19 m/s. Displacement: delta x = v0t + $(½)at^2 = 4(5) + 0.5(3)(5)^2$ = 57.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta $x=v0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 210

Problem: A force of 34 N pulls a 6 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 4.907 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 34(0.866). Then a = F_x/m = 29.444/6 = 4.907 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 211

Problem: A constant horizontal force of 17 N moves a box 5 m in the same direction. Find the work done.

Direct Answer: 85 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=17(5)=85 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 212

Problem: Find the momentum of a 4 kg object moving at 5 m/s.

Direct Answer: 20 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=5 m/s: p=4(5)=20 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 213

Problem: A force of 37 N is applied perpendicular to a wrench 0.3 m from the πvot. Find the torque magnitude.

Direct Answer: 11.1 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.3(37) = 11.1 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 214

Problem: A rotating object has rotational inertia 7 kg*m^2 and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 87.5 J

Detailed Work: Rotational kinetic energy is K_{rot} = (½)I $omega^2$. Substitute: K_{rot} = $0.5(7)(5)^2$ = 87.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 215

Problem: A mass-spring oscillator has mass 4 kg and spring constant 55 N/m. Find the period.

Direct Answer: 1.694 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=55: T = 2*π*√(4/55) = 1.694 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 216

Problem: An ideal incompressible fluid flows through a πpe. If A1=4 cm^2, v1=4 m/s, and A2=4 cm^2, find v2.

Direct Answer: 4 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 4(4)/4 = 4 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 217

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(9)/9.8) = 1.355 s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 218

Problem: A 3 kg object accelerates at 4 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 12 N

Detailed Work: Newton's second law gives F_{net} = ma. Substitute m=3 kg and a=4 m/s^2: F_{net} = 3(4) = 12 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 219

Problem: Find the kinetic energy of a 5 kg object moving at 10 m/s.

Direct Answer: 250 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = $0.5(5)(10)^2$ = 250 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 220

Problem: A 4 kg cart moving at 3 m/s sticks to a 5 kg cart at rest. Find their final speed.

Direct Answer: 1.333 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 4(3) + 5(0) = 12. Total mass = 9. Final speed v_f = 12/9 = 1.333 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 221

Problem: A net torque of 10 N*m acts on an object with rotational inertia 5 kg*m². Find the angular acceleration.

Direct Answer: 2 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 10/5 = 2 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 222

Problem: Find the angular momentum of a rigid body with rotational inertia 6 kg*m² and angular speed 2 rad/s.

Direct Answer: 12 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(2)=12 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 223

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=2.5 m and g=9.8 m/s²: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 224

Problem: Find the gauge pressure at a depth of 9 m in water. Use rho = 1000 kg/m³.

Direct Answer: 88200 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(9) = 88200 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 225

Problem: A cart starts with velocity 6 m/s and accelerates at 5 m/s² for 7 s. Find its final velocity and displacement.

Direct Answer: v = 41 m/s, displacement = 164.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+5(7) = 41 m/s. Displacement: delta x = v0t + (½)at² = 6(7) + 0.5(5)(7)² = 164.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 226

Problem: A 10 kg box slides on a level surface with kinetic friction coefficient 0.4. Find the friction force magnitude.

Direct Answer: 39.2 N

Detailed Work: On a level surface, normal force N = mg = 10(9.8). Kinetic friction is f_k = mu_k N = 0.4(10)(9.8) = 39.2 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 227

Problem: A 6 kg object is lifted 8 m. Find the gain in gravitational potential energy.

Direct Answer: 470.4 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 6(9.8)(8) = 470.4 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 228

Problem: Find the momentum of a 6 kg object moving at 7 m/s.

Direct Answer: 42 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=7 m/s: p=6(7)=42 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 229

Problem: A force of 39 N is applied perpendicular to a wrench 0.5 m from the πvot. Find the torque magnitude.

Direct Answer: 19.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.5(39) = 19.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 230

Problem: A rotating object has rotational inertia 2 kg*m^2 and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 49 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(2)$(7)^2$ = 49 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 231

Problem: A mass-spring oscillator has mass 6 kg and spring constant 65 N/m. Find the period.

Direct Answer: 1.909 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=65: T = 2*π*√(6/65) = 1.909 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 232

Problem: An object displaces 0.003 m^3 of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 233

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(11)/9.8) = 1.498 s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 234

Problem: A force of 40 N pulls a 9 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 3.849 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 40(0.866). Then a = F_x/m = 34.64/9 = 3.849 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 235

Problem: A constant horizontal force of 20 N moves a box 8 m in the same direction. Find the work done.

Direct Answer: 160 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=20(8)=160 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 236

Problem: A 1 kg cart moving at 5 m/s sticks to a 2 kg cart at rest. Find their final speed.

Direct Answer: 1.667 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 1(5) + 2(0) = 5. Total mass = 3. Final speed v_f = 5/3 = 1.667 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 237

Problem: A net torque of 2 N*m acts on an object with rotational inertia 1 kg*m^2. Find the angular acceleration.

Direct Answer: 2 rad/s^2

Detailed Work: Rotational Newton's second law is τ_{net} = I alpha. Solve alpha = tau/I = 2/1 = 2 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 238

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m^2 and angular speed 4 rad/s.

Direct Answer: 32 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(4)=32 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 239

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=3.5 m and g=9.8 m/s^2: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 240

Problem: An ideal incompressible fluid flows through a πpe. If A1=2 cm^2, v1=7 m/s, and A2=3 cm^2, find v2.

Direct Answer: 4.667 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 2(7)/3 = 4.667 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 241

Problem: A cart starts with velocity 8 m/s and accelerates at 2 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 14 m/s, displacement = 33 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+2(3) = 14 m/s. Displacement: delta x = v0t + (½)at^2 = 8(3) + 0.5(2)(3)^2 = 33 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 242

Problem: A 6 kg object accelerates at 2 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 12 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=6 kg and a=2 m/s²: F_net = 6(2) = 12 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 243

Problem: Find the kinetic energy of a 2 kg object moving at 3 m/s.

Direct Answer: 9 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(2)(3)^2 = 9$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 244

Problem: Find the momentum of a 8 kg object moving at 9 m/s.

Direct Answer: 72 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=9 m/s: p=8(9)=72 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 245

Problem: A force of 41 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 8.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(41) = 8.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 246

Problem: A rotating object has rotational inertia 4 $kg*m^2$ and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 162 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(4)(9)^2 = 162$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 247

Problem: A mass-spring oscillator has mass 2 kg and spring constant 25 N/m. Find the period.

Direct Answer: 1.777 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=25: T = 2*π*√(2/25) = 1.777 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 248

Problem: Find the gauge pressure at a depth of 2 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 19600 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(2) = 19600 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 249

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so t = √(2h/g) = √(2(5)/9.8) = 1.01 s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 250

Problem: A 5 kg box slides on a level surface with kinetic friction coefficient 0.25. Find the friction force magnitude.

Direct Answer: 12.25 N

Detailed Work: On a level surface, normal force N = mg = 5(9.8). Kinetic friction is $f_k = mu_k N$ = 0.25(5)(9.8) = 12.25 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 251

Problem: A 4 kg object is lifted 3 m. Find the gain in gravitational potential energy.

Direct Answer: 117.6 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 4(9.8)(3) = 117.6 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 252

Problem: A 3 kg cart moving at 7 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 3 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(7) + 4(0) = 21. Total mass = 7. Final speed v_f = 21/7 = 3 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 253

Problem: A net torque of 4 N*m acts on an object with rotational inertia 3 kg*m^2. Find the angular acceleration.

Direct Answer: 1.333 rad/s^2

Detailed Work: Rotational Newton's second law is tau_{net} = I alpha. Solve alpha = tau/I = 4/3 = 1.333 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 254

Problem: Find the angular momentum of a rigid body with rotational inertia 2 kg*m^2 and angular speed 6 rad/s.

Direct Answer: 12 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(6)=12 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 255

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=0.5 m and g=9.8 m/s^2: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 256

Problem: An object displaces 0.006 m^3 of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 257

Problem: A cart starts with velocity 2 m/s and accelerates at 4 m/s^2 for 5 s. Find its final velocity and displacement.

Direct Answer: v = 22 m/s, displacement = 60 m

Detailed Work: Use constant-acceleration equations. Final velocity: $v = v0 + at = 2+4(5) = 22$ m/s. Displacement: delta $x = v0t + (½)at^2 = 2(5) + 0.5(4)(5)^2 = 60$ m.

Memory Key: For constant acceleration, use $v=v0+at$ and delta $x=v0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 258

Problem: A force of 46 N pulls a 4 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 9.959 m/s^2

Detailed Work: Only the horizontal component accelerates the object: $F_x = F \cos 30 = 46(0.866)$. Then $a = F_x/m = 39.836/4 = 9.959$ m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 259

Problem: A constant horizontal force of 23 N moves a box 3 m in the same direction. Find the work done.

Direct Answer: 69 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=23(3)=69 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 260

Problem: Find the momentum of a 2 kg object moving at 11 m/s.

Direct Answer: 22 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=11 m/s: p=2(11)=22 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 261

Problem: A force of 43 N is applied perpendicular to a wrench 0.4 m from the πvot. Find the torque magnitude.

Direct Answer: 17.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.4(43) = 17.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 262

Problem: A rotating object has rotational inertia 6 $kg*m^2$ and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 27 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(6)(3)^2 = 27$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 263

Problem: A mass-spring oscillator has mass 4 kg and spring constant 35 N/m. Find the period.

Direct Answer: 2.124 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=35: T = 2*π*√(4/35) = 2.124 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 264

Problem: An ideal incompressible fluid flows through a πpe. If A1=5 cm^2, v1=2 m/s, and A2=2 cm^2, find v2.

Direct Answer: 5 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 5(2)/2 = 5 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 265

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(7)/9.8} = 1.195$ s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 266

Problem: A 9 kg object accelerates at 5 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 45 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=9 kg and a=5 m/s^2: F_net = 9(5) = 45 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 267

Problem: Find the kinetic energy of a 5 kg object moving at 6 m/s.

Direct Answer: 90 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(5)(6)^2 = 90$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 268

Problem: A 5 kg cart moving at 9 m/s sticks to a 6 kg cart at rest. Find their final speed.

Direct Answer: 4.091 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 5(9) + 6(0) = 45. Total mass = 11. Final speed v_f = 45/11 = 4.091 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 269

Problem: A net torque of 6 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 1.2 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 6/5 = 1.2 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 270

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m^2 and angular speed 8 rad/s.

Direct Answer: 32 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(8)=32 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 271

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{L/g}$. Substitute L=1.5 m and g=9.8 m/s^2: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 272

Problem: Find the gauge pressure at a depth of 5 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 49000 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(5) = 49000 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 273

Problem: A cart starts with velocity 4 m/s and accelerates at 1 m/s^2 for 7 s. Find its final velocity and displacement.

Direct Answer: v = 11 m/s, displacement = 52.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 4+1(7) = 11 m/s. Displacement: delta x = v0t + (½)at^2 = 4(7) + 0.5(1)$(7)^2$ = 52.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 274

Problem: A 8 kg box slides on a level surface with kinetic friction coefficient 0.1. Find the friction force magnitude.

Direct Answer: 7.84 N

Detailed Work: On a level surface, normal force N = mg = 8(9.8). Kinetic friction is f_k = mu_k N = 0.1(8)(9.8) = 7.84 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 275

Problem: A 2 kg object is lifted 6 m. Find the gain in gravitational potential energy.

Direct Answer: 117.6 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 2(9.8)(6) = 117.6 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 276

Problem: Find the momentum of a 4 kg object moving at 13 m/s.

Direct Answer: 52 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=13 m/s: p=4(13)=52 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 277

Problem: A force of 45 N is applied perpendicular to a wrench 0.1 m from the πvot. Find the torque magnitude.

Direct Answer: 4.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.1(45) = 4.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 278

Problem: A rotating object has rotational inertia 1 $kg*m^2$ and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 12.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(1)$(5)^2$ = 12.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 279

Problem: A mass-spring oscillator has mass 6 kg and spring constant 45 N/m. Find the period.

Direct Answer: 2.294 s

Detailed Work: For a mass-spring oscillator, $T = 2\pi\sqrt{m/k}$. Substitute m=6, k=45: $T = 2\pi\sqrt{6/45} = 2.294$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 280

Problem: An object displaces 0.009 m^3 of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: $F_b = \rho g V = 1000(9.8)(0.009) = 88.2$ N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 281

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(9)/9.8} = 1.355$ s. Horizontal speed is constant, so range = $v_x t$ = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 282

Problem: A force of 52 N pulls a 7 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 6.433 m/s^2

Detailed Work: Only the horizontal component accelerates the object: $F_x = F \cos 30 = 52(0.866)$. Then $a = F_x/m = 45.032/7 = 6.433$ m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 283

Problem: A constant horizontal force of 26 N moves a box 6 m in the same direction. Find the work done.

Direct Answer: 156 J

Detailed Work: Work is $W = Fd \cos\theta$. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=26(6)=156 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 284

Problem: A 2 kg cart moving at 4 m/s sticks to a 3 kg cart at rest. Find their final speed.

Direct Answer: 1.6 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 2(4) + 3(0) = 8. Total mass = 5. Final speed $v_f = 8/5 = 1.6$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 285

Problem: A net torque of 8 N*m acts on an object with rotational inertia 1 kg*m^2. Find the angular acceleration.

Direct Answer: 8 rad/s^2

Detailed Work: Rotational Newton's second law is $\tau_{net} = I\alpha$. Solve $\alpha = \tau/I = 8/1 = 8$ rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 286

Problem: Find the angular momentum of a rigid body with rotational inertia 6 kg*m^2 and angular speed 1 rad/s.

Direct Answer: 6 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(1)=6 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 287

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=2.5 m and g=9.8 m/s^2: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 288

Problem: An ideal incompressible fluid flows through a πpe. If A1=3 cm^2, v1=5 m/s, and A2=1 cm^2, find v2.

Direct Answer: 15 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 3(5)/1 = 15 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 289

Problem: A cart starts with velocity 6 m/s and accelerates at 3 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 15 m/s, displacement = 31.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+3(3) = 15 m/s. Displacement: delta x = v0t + (½)at^2 = 6(3) + 0.5(3)$(3)^2$ = 31.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 290

Problem: A 3 kg object accelerates at 3 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 9 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=3 kg and a=3 m/s^2: F_net = 3(3) = 9 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 291

Problem: Find the kinetic energy of a 2 kg object moving at 9 m/s.

Direct Answer: 81 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = 0.5(2)$(9)^2$ = 81 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 292

Problem: Find the momentum of a 6 kg object moving at 3 m/s.

Direct Answer: 18 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=3 m/s: p=6(3)=18 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 293

Problem: A force of 47 N is applied perpendicular to a wrench 0.3 m from the πvot. Find the torque magnitude.

Direct Answer: 14.1 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.3(47) = 14.1 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 294

Problem: A rotating object has rotational inertia 3 $kg*m^2$ and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 73.5 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(3)(7)^2 = 73.5$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 295

Problem: A mass-spring oscillator has mass 2 kg and spring constant 55 N/m. Find the period.

Direct Answer: 1.198 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=55: T = 2*π*√(2/55) = 1.198 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 296

Problem: Find the gauge pressure at a depth of 8 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 78400 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(8) = 78400 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 297

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so t = √(2h/g) = √(2(11)/9.8) = 1.498 s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 298

Problem: A 11 kg box slides on a level surface with kinetic friction coefficient 0.3. Find the friction force magnitude.

Direct Answer: 32.34 N

Detailed Work: On a level surface, normal force N = mg = 11(9.8). Kinetic friction is $f_k = mu_k$ N = 0.3(11)(9.8) = 32.34 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 299

Problem: A 5 kg object is lifted 9 m. Find the gain in gravitational potential energy.

Direct Answer: 441 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 5(9.8)(9) = 441 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 300

Problem: A 4 kg cart moving at 6 m/s sticks to a 5 kg cart at rest. Find their final speed.

Direct Answer: 2.667 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 4(6) + 5(0) = 24. Total mass = 9. Final speed v_f = 24/9 = 2.667 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 301

Problem: A net torque of 10 N*m acts on an object with rotational inertia 3 $kg*m^2$. Find the angular acceleration.

Direct Answer: 3.333 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 10/3 = 3.333 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 302

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m² and angular speed 3 rad/s.

Direct Answer: 24 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(3)=24 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 303

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=3.5 m and g=9.8 m/s²: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 304

Problem: An object displaces 0.003 m³ of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 305

Problem: A cart starts with velocity 8 m/s and accelerates at 5 m/s² for 5 s. Find its final velocity and displacement.

Direct Answer: v = 33 m/s, displacement = 102.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+5(5) = 33 m/s. Displacement: delta x = v0t + (½)at² = 8(5) + 0.5(5)(5)² = 102.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 306

Problem: A force of 58 N pulls a 10 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 5.023 m/s²

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 58(0.866). Then a = F_x/m = 50.228/10 = 5.023 m/s².

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 307

Problem: A constant horizontal force of 29 N moves a box 9 m in the same direction. Find the work done.

Direct Answer: 261 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=29(9)=261 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 308

Problem: Find the momentum of a 8 kg object moving at 5 m/s.

Direct Answer: 40 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=5 m/s: p=8(5)=40 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 309

Problem: A force of 49 N is applied perpendicular to a wrench 0.5 m from the πvot. Find the torque magnitude.

Direct Answer: 24.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.5(49) = 24.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 310

Problem: A rotating object has rotational inertia 5 kg*m^2 and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 202.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I omega^2. Substitute: K_rot = 0.5(5)(9)^2 = 202.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 311

Problem: A mass-spring oscillator has mass 4 kg and spring constant 65 N/m. Find the period.

Direct Answer: 1.559 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=65: T = 2*π*√(4/65) = 1.559 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 312

Problem: An ideal incompressible fluid flows through a πpe. If A1=6 cm^2, v1=8 m/s, and A2=4 cm^2, find v2.

Direct Answer: 12 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 6(8)/4 = 12 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 313

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(5)/9.8) = 1.01 s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 314

Problem: A 6 kg object accelerates at 1 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 6 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=6 kg and a=1 m/s^2: F_net = 6(1) = 6 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 315

Problem: Find the kinetic energy of a 5 kg object moving at 2 m/s.

Direct Answer: 10 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = 0.5(5)(2)^2 = 10 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 316

Problem: A 1 kg cart moving at 8 m/s sticks to a 2 kg cart at rest. Find their final speed.

Direct Answer: 2.667 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 1(8) + 2(0) = 8. Total mass = 3. Final speed $v_f = 8/3 = 2.667$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 317

Problem: A net torque of 2 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 0.4 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 2/5 = 0.4 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 318

Problem: Find the angular momentum of a rigid body with rotational inertia 2 kg*m^2 and angular speed 5 rad/s.

Direct Answer: 10 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(5)=10 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 319

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=0.5 m and g=9.8 m/s^2: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 320

Problem: Find the gauge pressure at a depth of 1 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 9800 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(1) = 9800 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 321

Problem: A cart starts with velocity 2 m/s and accelerates at 2 m/s^2 for 7 s. Find its final velocity and displacement.

Direct Answer: v = 16 m/s, displacement = 63 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 2+2(7) = 16 m/s. Displacement: delta x = v0t + (½)at^2 = 2(7) + 0.5(2)(7)^2 = 63 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 322

Problem: A 6 kg box slides on a level surface with kinetic friction coefficient 0.2. Find the friction force magnitude.

Direct Answer: 11.76 N

Detailed Work: On a level surface, normal force N = mg = 6(9.8). Kinetic friction is f_k = mu_k N = 0.2(6)(9.8) = 11.76 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 323

Problem: A 3 kg object is lifted 4 m. Find the gain in gravitational potential energy.

Direct Answer: 117.6 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 3(9.8)(4) = 117.6 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 324

Problem: Find the momentum of a 2 kg object moving at 7 m/s.

Direct Answer: 14 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=7 m/s: p=2(7)=14 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 325

Problem: A force of 11 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 2.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(11) = 2.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 326

Problem: A rotating object has rotational inertia 7 $kg*m^2$ and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 31.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = $0.5(7)(3)^2$ = 31.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 327

Problem: A mass-spring oscillator has mass 6 kg and spring constant 25 N/m. Find the period.

Direct Answer: 3.078 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=25: T = 2*π*√(6/25) = 3.078 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 328

Problem: An object displaces 0.006 m^3 of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 329

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(7)/9.8) = 1.195 s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 330

Problem: A force of 24 N pulls a 5 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 4.157 m/s^2

Detailed Work: Only the horizontal component accelerates the object: $F_x = F \cos 30 = 24(0.866)$. Then $a = F_x/m = 20.784/5 = 4.157$ m/s².

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 331

Problem: A constant horizontal force of 12 N moves a box 4 m in the same direction. Find the work done.

Direct Answer: 48 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=12(4)=48 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 332

Problem: A 3 kg cart moving at 3 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 1.286 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(3) + 4(0) = 9. Total mass = 7. Final speed $v_f = 9/7 = 1.286$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 333

Problem: A net torque of 4 N*m acts on an object with rotational inertia 1 kg*m². Find the angular acceleration.

Direct Answer: 4 rad/s²

Detailed Work: Rotational Newton's second law is $\tau_{net} = I \alpha$. Solve alpha = tau/I = 4/1 = 4 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 334

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m² and angular speed 7 rad/s.

Direct Answer: 28 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(7)=28 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 335

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=1.5 m and g=9.8 m/s²: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 336

Problem: An ideal incompressible fluid flows through a πpe. If A1=4 cm², v1=3 m/s, and A2=3 cm², find v2.

Direct Answer: 4 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 4(3)/3 = 4 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 337

Problem: A cart starts with velocity 4 m/s and accelerates at 4 m/s² for 3 s. Find its final velocity and displacement.

Direct Answer: v = 16 m/s, displacement = 30 m

Detailed Work: Use constant-acceleration equations. Final velocity: $v = v_0 + at = 4+4(3) = 16$ m/s. Displacement: $\Delta x = v_0 t + (\frac{1}{2})at^2 = 4(3) + 0.5(4)(3)^2 = 30$ m.

Memory Key: For constant acceleration, use $v=v_0+at$ and delta $x=v_0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 338

Problem: A 9 kg object accelerates at 4 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 36 N

Detailed Work: Newton's second law gives $F_{net} = ma$. Substitute m=9 kg and a=4 m/s^2: $F_{net} = 9(4) = 36$ N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 339

Problem: Find the kinetic energy of a 2 kg object moving at 5 m/s.

Direct Answer: 25 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(2)(5)^2 = 25$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 340

Problem: Find the momentum of a 4 kg object moving at 9 m/s.

Direct Answer: 36 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=9 m/s: p=4(9)=36 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 341

Problem: A force of 13 N is applied perpendicular to a wrench 0.4 m from the πvot. Find the torque magnitude.

Direct Answer: 5.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.4(13) = 5.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 342

Problem: A rotating object has rotational inertia 2 kg*m^2 and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 25 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(2)(5)^2 = 25$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 343

Problem: A mass-spring oscillator has mass 2 kg and spring constant 35 N/m. Find the period.

Direct Answer: 1.502 s

Detailed Work: For a mass-spring oscillator, $T = 2*\pi*\sqrt{(m/k)}$. Substitute m=2, k=35: $T = 2*\pi*\sqrt{(2/35)} = 1.502$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 344

Problem: Find the gauge pressure at a depth of 4 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 39200 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(4) = 39200 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 345

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(9)/9.8} = 1.355$ s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 346

Problem: A 9 kg box slides on a level surface with kinetic friction coefficient 0.4. Find the friction force magnitude.

Direct Answer: 35.28 N

Detailed Work: On a level surface, normal force N = mg = 9(9.8). Kinetic friction is f_k = mu_k N = 0.4(9)(9.8) = 35.28 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 347

Problem: A 6 kg object is lifted 7 m. Find the gain in gravitational potential energy.

Direct Answer: 411.6 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 6(9.8)(7) = 411.6 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 348

Problem: A 5 kg cart moving at 5 m/s sticks to a 6 kg cart at rest. Find their final speed.

Direct Answer: 2.273 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 5(5) + 6(0) = 25. Total mass = 11. Final speed v_f = 25/11 = 2.273 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 349

Problem: A net torque of 6 N*m acts on an object with rotational inertia 3 $kg*m^2$. Find the angular acceleration.

Direct Answer: 2 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 6/3 = 2 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 350

Problem: Find the angular momentum of a rigid body with rotational inertia 6 $kg*m^2$ and angular speed 9 rad/s.

Direct Answer: 54 $kg*m^2/s$

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(9)=54 $kg*m^2/s$.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 351

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{L/g}$. Substitute L=2.5 m and g=9.8 m/s^2: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 352

Problem: An object displaces 0.009 m^3 of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: $F_b = \rho g V = 1000(9.8)(0.009) = 88.2$ N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 353

Problem: A cart starts with velocity 6 m/s and accelerates at 1 m/s² for 5 s. Find its final velocity and displacement.

Direct Answer: v = 11 m/s, displacement = 42.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: $v = v_0 + at = 6+1(5) = 11$ m/s. Displacement: $\Delta x = v_0 t + (½)at^2 = 6(5) + 0.5(1)(5)^2 = 42.5$ m.

Memory Key: For constant acceleration, use $v=v_0+at$ and $\Delta x=v_0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 354

Problem: A force of 30 N pulls a 8 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 3.248 m/s²

Detailed Work: Only the horizontal component accelerates the object: $F_x = F \cos 30 = 30(0.866)$. Then $a = F_x/m = 25.98/8 = 3.248$ m/s².

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 355

Problem: A constant horizontal force of 15 N moves a box 7 m in the same direction. Find the work done.

Direct Answer: 105 J

Detailed Work: Work is $W = Fd \cos \theta$. Since force and displacement are in the same direction, $\theta=0$ and $\cos 0=1$. $W=15(7)=105$ J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 356

Problem: Find the momentum of a 6 kg object moving at 11 m/s.

Direct Answer: 66 kg*m/s

Detailed Work: Momentum is $p = mv$. Substitute m=6 kg and v=11 m/s: $p=6(11)=66$ kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 357

Problem: A force of 15 N is applied perpendicular to a wrench 0.1 m from the πvot. Find the torque magnitude.

Direct Answer: 1.5 N*m

Detailed Work: Torque magnitude is $\tau = rF \sin \theta$. Because the force is perpendicular, $\sin 90 = 1$. $\tau = 0.1(15) = 1.5$ N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 358

Problem: A rotating object has rotational inertia 4 kg*m² and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 98 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I \omega^2$. Substitute: $K_{rot} = 0.5(4)(7)^2 = 98$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 359

Problem: A mass-spring oscillator has mass 4 kg and spring constant 45 N/m. Find the period.

Direct Answer: 1.873 s

Detailed Work: For a mass-spring oscillator, $T = 2\pi\sqrt{m/k}$. Substitute m=4, k=45: $T = 2\pi\sqrt{4/45} = 1.873$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 360

Problem: An ideal incompressible fluid flows through a πpe. If A1=2 cm^2, v1=6 m/s, and A2=2 cm^2, find v2.

Direct Answer: 6 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 2(6)/2 = 6 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 361

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(11)/9.8) = 1.498 s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 362

Problem: A 3 kg object accelerates at 2 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 6 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=3 kg and a=2 m/s^2: F_net = 3(2) = 6 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 363

Problem: Find the kinetic energy of a 5 kg object moving at 8 m/s.

Direct Answer: 160 J

Detailed Work: Kinetic energy is K = (½)mv^2. Substitute: K = $0.5(5)(8)^2$ = 160 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 364

Problem: A 2 kg cart moving at 7 m/s sticks to a 3 kg cart at rest. Find their final speed.

Direct Answer: 2.8 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 2(7) + 3(0) = 14. Total mass = 5. Final speed v_f = 14/5 = 2.8 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 365

Problem: A net torque of 8 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 1.6 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 8/5 = 1.6 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 366

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m^2 and angular speed 2 rad/s.

Direct Answer: 16 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(2)=16 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 367

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=3.5 m and g=9.8 m/s^2: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 368

Problem: Find the gauge pressure at a depth of 7 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 68600 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(7) = 68600 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 369

Problem: A cart starts with velocity 8 m/s and accelerates at 3 m/s^2 for 7 s. Find its final velocity and displacement.

Direct Answer: v = 29 m/s, displacement = 129.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+3(7) = 29 m/s. Displacement: delta x = v0t + (½)at^2 = 8(7) + 0.5(3)(7)^2 = 129.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 370

Problem: A 12 kg box slides on a level surface with kinetic friction coefficient 0.25. Find the friction force magnitude.

Direct Answer: 29.4 N

Detailed Work: On a level surface, normal force N = mg = 12(9.8). Kinetic friction is f_k = mu_k N = 0.25(12)(9.8) = 29.4 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 371

Problem: A 4 kg object is lifted 10 m. Find the gain in gravitational potential energy.

Direct Answer: 392 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 4(9.8)(10) = 392 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 372

Problem: Find the momentum of a 8 kg object moving at 13 m/s.

Direct Answer: 104 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=13 m/s: p=8(13)=104 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 373

Problem: A force of 17 N is applied perpendicular to a wrench 0.3 m from the πvot. Find the torque magnitude.

Direct Answer: 5.1 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.3(17) = 5.1 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 374

Problem: A rotating object has rotational inertia 6 kg*m^2 and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 243 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I omega^2. Substitute: K_rot = 0.5(6)(9)^2 = 243 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 375

Problem: A mass-spring oscillator has mass 6 kg and spring constant 55 N/m. Find the period.

Direct Answer: 2.075 s

Detailed Work: For a mass-spring oscillator, $T = 2*\pi*\sqrt{(m/k)}$. Substitute m=6, k=55: $T = 2*\pi*\sqrt{(6/55)} = 2.075$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 376

Problem: An object displaces 0.003 m^3 of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 377

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{(2h/g)} = \sqrt{(2(5)/9.8)} = 1.01$ s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 378

Problem: A force of 36 N pulls a 3 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 10.392 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 36(0.866). Then a = F_x/m = 31.176/3 = 10.392 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 379

Problem: A constant horizontal force of 18 N moves a box 2 m in the same direction. Find the work done.

Direct Answer: 36 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=18(2)=36 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 380

Problem: A 4 kg cart moving at 9 m/s sticks to a 5 kg cart at rest. Find their final speed.

Direct Answer: 4 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 4(9) + 5(0) = 36. Total mass = 9. Final speed v_f = 36/9 = 4 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 381

Problem: A net torque of 10 N*m acts on an object with rotational inertia 1 kg*m^2. Find the angular acceleration.

Direct Answer: 10 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 10/1 = 10 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 382

Problem: Find the angular momentum of a rigid body with rotational inertia 2 $kg*m^2$ and angular speed 4 rad/s.

Direct Answer: 8 $kg*m^2/s$

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(4)=8 $kg*m^2/s$.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 383

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=0.5 m and g=9.8 m/s^2: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 384

Problem: An ideal incompressible fluid flows through a πpe. If A1=5 cm^2, v1=1 m/s, and A2=1 cm^2, find v2.

Direct Answer: 5 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 5(1)/1 = 5 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 385

Problem: A cart starts with velocity 2 m/s and accelerates at 5 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 17 m/s, displacement = 28.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 2+5(3) = 17 m/s. Displacement: delta x = v0t + $(½)at^2$ = 2(3) + $0.5(5)(3)^2$ = 28.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+$0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 386

Problem: A 6 kg object accelerates at 5 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 30 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=6 kg and a=5 m/s^2: F_net = 6(5) = 30 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 387

Problem: Find the kinetic energy of a 2 kg object moving at 11 m/s.

Direct Answer: 121 J

Detailed Work: Kinetic energy is K = $(½)mv^2$. Substitute: K = $0.5(2)(11)^2$ = 121 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 388

Problem: Find the momentum of a 2 kg object moving at 3 m/s.

Direct Answer: 6 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=3 m/s: p=2(3)=6 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 389

Problem: A force of 19 N is applied perpendicular to a wrench 0.5 m from the πvot. Find the torque magnitude.

Direct Answer: 9.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.5(19) = 9.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 390

Problem: A rotating object has rotational inertia 1 kg*m^2 and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 4.5 J

Detailed Work: Rotational kinetic energy is K_{rot} = (½)I omega^2. Substitute: K_{rot} = 0.5(1)(3)^2 = 4.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 391

Problem: A mass-spring oscillator has mass 2 kg and spring constant 65 N/m. Find the period.

Direct Answer: 1.102 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=65: T = 2*π*√(2/65) = 1.102 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 392

Problem: Find the gauge pressure at a depth of 10 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 98000 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(10) = 98000 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 393

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(7)/9.8) = 1.195 s. Horizontal speed is constant, so range = vx*t = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 394

Problem: A 7 kg box slides on a level surface with kinetic friction coefficient 0.1. Find the friction force magnitude.

Direct Answer: 6.86 N

Detailed Work: On a level surface, normal force N = mg = 7(9.8). Kinetic friction is f_k = mu_k N = 0.1(7)(9.8) = 6.86 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 395

Problem: A 2 kg object is lifted 5 m. Find the gain in gravitational potential energy.

Direct Answer: 98 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 2(9.8)(5) = 98 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 396

Problem: A 1 kg cart moving at 4 m/s sticks to a 2 kg cart at rest. Find their final speed.

Direct Answer: 1.333 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 1(4) + 2(0) = 4. Total mass = 3. Final speed $v_f = 4/3 = 1.333$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 397

Problem: A net torque of 2 N*m acts on an object with rotational inertia 3 kg*m^2. Find the angular acceleration.

Direct Answer: 0.667 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 2/3 = 0.667 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 398

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m^2 and angular speed 6 rad/s.

Direct Answer: 24 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(6)=24 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 399

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=1.5 m and g=9.8 m/s^2: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 400

Problem: An object displaces 0.006 m^3 of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 401

Problem: A cart starts with velocity 4 m/s and accelerates at 2 m/s^2 for 5 s. Find its final velocity and displacement.

Direct Answer: v = 14 m/s, displacement = 45 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 4+2(5) = 14 m/s. Displacement: delta x = v0t + (½)at^2 = 4(5) + 0.5(2)(5)^2 = 45 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 402

Problem: A force of 42 N pulls a 6 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 6.062 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 42(0.866). Then a = F_x/m = 36.372/6 = 6.062 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 403

Problem: A constant horizontal force of 21 N moves a box 5 m in the same direction. Find the work done.

Direct Answer: 105 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=21(5)=105 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 404

Problem: Find the momentum of a 4 kg object moving at 5 m/s.

Direct Answer: 20 kg*m/s

Detailed Work: Momentum is $p = mv$. Substitute m=4 kg and v=5 m/s: p=4(5)=20 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 405

Problem: A force of 21 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 4.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(21) = 4.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 406

Problem: A rotating object has rotational inertia 3 kg*m^2 and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 37.5 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ omega^2$. Substitute: $K_{rot} = 0.5(3)(5)^2 = 37.5$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 407

Problem: A mass-spring oscillator has mass 4 kg and spring constant 25 N/m. Find the period.

Direct Answer: 2.513 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=4, k=25: T = 2*π*√(4/25) = 2.513 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 408

Problem: An ideal incompressible fluid flows through a πpe. If A1=3 cm^2, v1=4 m/s, and A2=4 cm^2, find v2.

Direct Answer: 3 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 3(4)/4 = 3 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 409

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so t = √(2h/g) = √(2(9)/9.8) = 1.355 s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 410

Problem: A 9 kg object accelerates at 3 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 27 N

Detailed Work: Newton's second law gives $F_{net} = ma$. Substitute m=9 kg and a=3 m/s^2: $F_{net} = 9(3) = 27$ N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 411

Problem: Find the kinetic energy of a 5 kg object moving at 4 m/s.

Direct Answer: 40 J

Detailed Work: Kinetic energy is $K = (\frac{1}{2})mv^2$. Substitute: $K = 0.5(5)(4)^2 = 40$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 412

Problem: A 3 kg cart moving at 6 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 2.571 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(6) + 4(0) = 18. Total mass = 7. Final speed $v_f = 18/7 = 2.571$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 413

Problem: A net torque of 4 N*m acts on an object with rotational inertia 5 kg*m^2. Find the angular acceleration.

Direct Answer: 0.8 rad/s^2

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 4/5 = 0.8 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 414

Problem: Find the angular momentum of a rigid body with rotational inertia 6 kg*m^2 and angular speed 8 rad/s.

Direct Answer: 48 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(8)=48 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 415

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=2.5 m and g=9.8 m/s^2: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 416

Problem: Find the gauge pressure at a depth of 3 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 29400 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(3) = 29400 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 417

Problem: A cart starts with velocity 6 m/s and accelerates at 4 m/s^2 for 7 s. Find its final velocity and displacement.

Direct Answer: v = 34 m/s, displacement = 140 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+4(7) = 34 m/s. Displacement: delta x = v0t + $(\frac{1}{2})at^2 = 6(7) + 0.5(4)(7)^2 = 140$ m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 418

Problem: A 10 kg box slides on a level surface with kinetic friction coefficient 0.3. Find the friction force magnitude.

Direct Answer: 29.4 N

Detailed Work: On a level surface, normal force N = mg = 10(9.8). Kinetic friction is f_k = mu_k N = 0.3(10)(9.8) = 29.4 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 419

Problem: A 5 kg object is lifted 8 m. Find the gain in gravitational potential energy.

Direct Answer: 392 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 5(9.8)(8) = 392 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 420

Problem: Find the momentum of a 6 kg object moving at 7 m/s.

Direct Answer: 42 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=7 m/s: p=6(7)=42 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 421

Problem: A force of 23 N is applied perpendicular to a wrench 0.4 m from the πvot. Find the torque magnitude.

Direct Answer: 9.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.4(23) = 9.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 422

Problem: A rotating object has rotational inertia 5 kg*m^2 and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 122.5 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(5)$(7)^2$ = 122.5 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 423

Problem: A mass-spring oscillator has mass 6 kg and spring constant 35 N/m. Find the period.

Direct Answer: 2.601 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=35: T = 2*π*√(6/35) = 2.601 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 424

Problem: An object displaces 0.009 m^3 of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.009) = 88.2 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 425

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(11)/9.8) = 1.498 s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 426

Problem: A force of 48 N pulls a 9 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 4.619 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 48(0.866). Then a = F_x/m = 41.568/9 = 4.619 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 427

Problem: A constant horizontal force of 24 N moves a box 8 m in the same direction. Find the work done.

Direct Answer: 192 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=24(8)=192 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 428

Problem: A 5 kg cart moving at 8 m/s sticks to a 6 kg cart at rest. Find their final speed.

Direct Answer: 3.636 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 5(8) + 6(0) = 40. Total mass = 11. Final speed v_f = 40/11 = 3.636 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 429

Problem: A net torque of 6 N*m acts on an object with rotational inertia 1 kg*m^2. Find the angular acceleration.

Direct Answer: 6 rad/s^2

Detailed Work: Rotational Newton's second law is tau_{net} = I alpha. Solve alpha = tau/I = 6/1 = 6 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 430

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m^2 and angular speed 1 rad/s.

Direct Answer: 8 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(1)=8 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 431

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=3.5 m and g=9.8 m/s^2: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 432

Problem: An ideal incompressible fluid flows through a πpe. If A1=6 cm^2, v1=7 m/s, and A2=3 cm^2, find v2.

Direct Answer: 14 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 6(7)/3 = 14 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 433

Problem: A cart starts with velocity 8 m/s and accelerates at 1 m/s^2 for 3 s. Find its final velocity and displacement.

Direct Answer: v = 11 m/s, displacement = 28.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: $v = v0 + at = 8+1(3) = 11$ m/s. Displacement: $\Delta x = v0t + (½)at^2 = 8(3) + 0.5(1)(3)^2 = 28.5$ m.

Memory Key: For constant acceleration, use $v=v0+at$ and $\Delta x=v0t+0.5at^2$.

Unit 2: Force and Translational Dynamics

Question 434

Problem: A 3 kg object accelerates at 1 m/s^2 on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 3 N

Detailed Work: Newton's second law gives $F_{net} = ma$. Substitute m=3 kg and a=1 m/s^2: $F_{net} = 3(1) = 3$ N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 435

Problem: Find the kinetic energy of a 2 kg object moving at 7 m/s.

Direct Answer: 49 J

Detailed Work: Kinetic energy is $K = (½)mv^2$. Substitute: $K = 0.5(2)(7)^2 = 49$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 436

Problem: Find the momentum of a 8 kg object moving at 9 m/s.

Direct Answer: 72 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=9 m/s: p=8(9)=72 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 437

Problem: A force of 25 N is applied perpendicular to a wrench 0.1 m from the πvot. Find the torque magnitude.

Direct Answer: 2.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.1(25) = 2.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 438

Problem: A rotating object has rotational inertia 7 $kg*m^2$ and angular speed 9 rad/s. Find its rotational kinetic energy.

Direct Answer: 283.5 J

Detailed Work: Rotational kinetic energy is $K_{rot} = (½)I\ \omega^2$. Substitute: $K_{rot} = 0.5(7)(9)^2 = 283.5$ J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 439

Problem: A mass-spring oscillator has mass 2 kg and spring constant 45 N/m. Find the period.

Direct Answer: 1.325 s

Detailed Work: For a mass-spring oscillator, $T = 2*\pi*\sqrt{(m/k)}$. Substitute m=2, k=45: $T = 2*\pi*\sqrt{(2/45)} = 1.325$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 440

Problem: Find the gauge pressure at a depth of 6 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 58800 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(6) = 58800 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 441

Problem: A ball rolls horizontally off a table at 4 m/s from a height of 5 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.01 s, range = 4.041 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(5)/9.8} = 1.01$ s. Horizontal speed is constant, so range = vx*t = 4(1.01) = 4.041 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 442

Problem: A 5 kg box slides on a level surface with kinetic friction coefficient 0.2. Find the friction force magnitude.

Direct Answer: 9.8 N

Detailed Work: On a level surface, normal force N = mg = 5(9.8). Kinetic friction is $f_k = \mu_k N$ = 0.2(5)(9.8) = 9.8 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 443

Problem: A 3 kg object is lifted 3 m. Find the gain in gravitational potential energy.

Direct Answer: 88.2 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 3(9.8)(3) = 88.2 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 444

Problem: A 2 kg cart moving at 3 m/s sticks to a 3 kg cart at rest. Find their final speed.

Direct Answer: 1.2 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 2(3) + 3(0) = 6. Total mass = 5. Final speed v_f = 6/5 = 1.2 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 445

Problem: A net torque of 8 N*m acts on an object with rotational inertia 3 kg*m^2. Find the angular acceleration.

Direct Answer: 2.667 rad/s^2

Detailed Work: Rotational Newton's second law is τ_{net} = I alpha. Solve alpha = tau/I = 8/3 = 2.667 rad/s^2.

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 446

Problem: Find the angular momentum of a rigid body with rotational inertia 2 kg*m^2 and angular speed 3 rad/s.

Direct Answer: 6 kg*m^2/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=2(3)=6 kg*m^2/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 447

Problem: A simple pendulum has length 0.5 m. Estimate its period for small oscillations.

Direct Answer: 1.419 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=0.5 m and g=9.8 m/s^2: T = 1.419 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 448

Problem: An object displaces 0.003 m^3 of water. Find the buoyant force.

Direct Answer: 29.4 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.003) = 29.4 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 449

Problem: A cart starts with velocity 2 m/s and accelerates at 3 m/s^2 for 5 s. Find its final velocity and displacement.

Direct Answer: v = 17 m/s, displacement = 47.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 2+3(5) = 17 m/s. Displacement: delta x = v0t + (½)at^2 = 2(5) + 0.5(3)$(5)^2$ = 47.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at^2.

Unit 2: Force and Translational Dynamics

Question 450

Problem: A force of 54 N pulls a 4 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 11.691 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 54(0.866). Then a = F_x/m = 46.764/4 = 11.691 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 451

Problem: A constant horizontal force of 27 N moves a box 3 m in the same direction. Find the work done.

Direct Answer: 81 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=27(3)=81 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 452

Problem: Find the momentum of a 2 kg object moving at 11 m/s.

Direct Answer: 22 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=2 kg and v=11 m/s: p=2(11)=22 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 453

Problem: A force of 27 N is applied perpendicular to a wrench 0.3 m from the πvot. Find the torque magnitude.

Direct Answer: 8.1 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.3(27) = 8.1 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 454

Problem: A rotating object has rotational inertia 2 kg*m^2 and angular speed 3 rad/s. Find its rotational kinetic energy.

Direct Answer: 9 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(2)$(3)^2$ = 9 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 455

Problem: A mass-spring oscillator has mass 4 kg and spring constant 55 N/m. Find the period.

Direct Answer: 1.694 s

Detailed Work: For a mass-spring oscillator, $T = 2\pi\sqrt{m/k}$. Substitute m=4, k=55: $T = 2\pi\sqrt{4/55} = 1.694$ s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 456

Problem: An ideal incompressible fluid flows through a πpe. If $A_1=4$ cm², $v_1=2$ m/s, and $A_2=2$ cm², find v_2.

Direct Answer: 4 m/s

Detailed Work: Continuity gives $A_1v_1 = A_2v_2$. Solve $v_2 = A_1v_1/A_2 = 4(2)/2 = 4$ m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 457

Problem: A ball rolls horizontally off a table at 6 m/s from a height of 7 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.195 s, range = 7.171 m

Detailed Work: Vertical motion determines time: $h = 0.5gt^2$, so $t = \sqrt{2h/g} = \sqrt{2(7)/9.8} = 1.195$ s. Horizontal speed is constant, so range = $v_x t$ = 6(1.195) = 7.171 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 458

Problem: A 6 kg object accelerates at 4 m/s² on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 24 N

Detailed Work: Newton's second law gives $F_{net} = ma$. Substitute m=6 kg and a=4 m/s²: $F_{net} = 6(4) = 24$ N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 459

Problem: Find the kinetic energy of a 5 kg object moving at 10 m/s.

Direct Answer: 250 J

Detailed Work: Kinetic energy is $K = (\frac{1}{2})mv^2$. Substitute: $K = 0.5(5)(10)^2 = 250$ J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 460

Problem: A 4 kg cart moving at 5 m/s sticks to a 5 kg cart at rest. Find their final speed.

Direct Answer: 2.222 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 4(5) + 5(0) = 20. Total mass = 9. Final speed $v_f = 20/9 = 2.222$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 461

Problem: A net torque of 10 N*m acts on an object with rotational inertia 5 kg*m². Find the angular acceleration.

Direct Answer: 2 rad/s²

Detailed Work: Rotational Newton's second law is $\tau_{net} = I\alpha$. Solve $\alpha = \tau/I = 10/5 = 2$ rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 462

Problem: Find the angular momentum of a rigid body with rotational inertia 4 kg*m² and angular speed 5 rad/s.

Direct Answer: 20 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=4(5)=20 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 463

Problem: A simple pendulum has length 1.5 m. Estimate its period for small oscillations.

Direct Answer: 2.458 s

Detailed Work: For a simple pendulum, $T = 2*\pi*\sqrt{(L/g)}$. Substitute L=1.5 m and g=9.8 m/s²: T = 2.458 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 464

Problem: Find the gauge pressure at a depth of 9 m in water. Use rho = 1000 kg/m³.

Direct Answer: 88200 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(9) = 88200 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 465

Problem: A cart starts with velocity 4 m/s and accelerates at 5 m/s² for 7 s. Find its final velocity and displacement.

Direct Answer: v = 39 m/s, displacement = 150.5 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 4+5(7) = 39 m/s. Displacement: delta x = v0t + (½)at² = 4(7) + 0.5(5)(7)² = 150.5 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 466

Problem: A 8 kg box slides on a level surface with kinetic friction coefficient 0.4. Find the friction force magnitude.

Direct Answer: 31.36 N

Detailed Work: On a level surface, normal force N = mg = 8(9.8). Kinetic friction is f_k = mu_k N = 0.4(8)(9.8) = 31.36 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 467

Problem: A 6 kg object is lifted 6 m. Find the gain in gravitational potential energy.

Direct Answer: 352.8 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 6(9.8)(6) = 352.8 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 468

Problem: Find the momentum of a 4 kg object moving at 13 m/s.

Direct Answer: 52 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=4 kg and v=13 m/s: p=4(13)=52 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 469

Problem: A force of 29 N is applied perpendicular to a wrench 0.5 m from the πvot. Find the torque magnitude.

Direct Answer: 14.5 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.5(29) = 14.5 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 470

Problem: A rotating object has rotational inertia 4 kg*m^2 and angular speed 5 rad/s. Find its rotational kinetic energy.

Direct Answer: 50 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = 0.5(4)$(5)^2$ = 50 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 471

Problem: A mass-spring oscillator has mass 6 kg and spring constant 65 N/m. Find the period.

Direct Answer: 1.909 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=6, k=65: T = 2*π*√(6/65) = 1.909 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 472

Problem: An object displaces 0.006 m^3 of water. Find the buoyant force.

Direct Answer: 58.8 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.006) = 58.8 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 473

Problem: A ball rolls horizontally off a table at 8 m/s from a height of 9 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.355 s, range = 10.842 m

Detailed Work: Vertical motion determines time: h = 0.5gt^2, so t = √(2h/g) = √(2(9)/9.8) = 1.355 s. Horizontal speed is constant, so range = vx*t = 8(1.355) = 10.842 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 474

Problem: A force of 20 N pulls a 7 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 2.474 m/s^2

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 20(0.866). Then a = F_x/m = 17.32/7 = 2.474 m/s^2.

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 475

Problem: A constant horizontal force of 10 N moves a box 6 m in the same direction. Find the work done.

Direct Answer: 60 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=10(6)=60 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 476

Problem: A 1 kg cart moving at 7 m/s sticks to a 2 kg cart at rest. Find their final speed.

Direct Answer: 2.333 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 1(7) + 2(0) = 7. Total mass = 3. Final speed v_f = 7/3 = 2.333 m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 477

Problem: A net torque of 2 N*m acts on an object with rotational inertia 1 kg*m². Find the angular acceleration.

Direct Answer: 2 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 2/1 = 2 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 478

Problem: Find the angular momentum of a rigid body with rotational inertia 6 kg*m² and angular speed 7 rad/s.

Direct Answer: 42 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=6(7)=42 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 479

Problem: A simple pendulum has length 2.5 m. Estimate its period for small oscillations.

Direct Answer: 3.173 s

Detailed Work: For a simple pendulum, T = 2*π*√(L/g). Substitute L=2.5 m and g=9.8 m/s²: T = 3.173 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 480

Problem: An ideal incompressible fluid flows through a πpe. If A1=2 cm², v1=5 m/s, and A2=1 cm², find v2.

Direct Answer: 10 m/s

Detailed Work: Continuity gives A1v1 = A2v2. Solve v2 = A1v1/A2 = 2(5)/1 = 10 m/s.

Memory Key: For incompressible flow, smaller area means faster speed.

Unit 1: Kinematics

Question 481

Problem: A cart starts with velocity 6 m/s and accelerates at 2 m/s² for 3 s. Find its final velocity and displacement.

Direct Answer: v = 12 m/s, displacement = 27 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 6+2(3) = 12 m/s. Displacement: delta x = v0t + (½)at² = 6(3) + 0.5(2)(3)² = 27 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 482

Problem: A 9 kg object accelerates at 2 m/s² on a horizontal surface with negligible friction. What net force acts on it?

Direct Answer: 18 N

Detailed Work: Newton's second law gives F_net = ma. Substitute m=9 kg and a=2 m/s²: F_net = 9(2) = 18 N.

Memory Key: Net force equals mass times acceleration.

Unit 3: Work, Energy, and Power

Question 483

Problem: Find the kinetic energy of a 2 kg object moving at 3 m/s.

Direct Answer: 9 J

Detailed Work: Kinetic energy is K = (½)mv². Substitute: K = 0.5(2)(3)² = 9 J.

Memory Key: Kinetic energy depends on v squared.

Unit 4: Linear Momentum

Question 484

Problem: Find the momentum of a 6 kg object moving at 3 m/s.

Direct Answer: 18 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=6 kg and v=3 m/s: p=6(3)=18 kg*m/s.

Memory Key: Momentum is mass times velocity.

Unit 5: Torque and Rotational Dynamics

Question 485

Problem: A force of 31 N is applied perpendicular to a wrench 0.2 m from the πvot. Find the torque magnitude.

Direct Answer: 6.2 N*m

Detailed Work: Torque magnitude is tau = rF sin theta. Because the force is perpendicular, sin 90 = 1. tau = 0.2(31) = 6.2 N*m.

Memory Key: Maximum torque occurs when the force is perpendicular to the lever arm.

Unit 6: Energy and Momentum of Rotating Systems

Question 486

Problem: A rotating object has rotational inertia 6 $kg*m^2$ and angular speed 7 rad/s. Find its rotational kinetic energy.

Direct Answer: 147 J

Detailed Work: Rotational kinetic energy is K_rot = (½)I $omega^2$. Substitute: K_rot = $0.5(6)(7)^2$ = 147 J.

Memory Key: Rotational kinetic energy mirrors translational kinetic energy.

Unit 7: Oscillations

Question 487

Problem: A mass-spring oscillator has mass 2 kg and spring constant 25 N/m. Find the period.

Direct Answer: 1.777 s

Detailed Work: For a mass-spring oscillator, T = 2*π*√(m/k). Substitute m=2, k=25: T = 2*π*√(2/25) = 1.777 s.

Memory Key: Spring period increases with mass and decreases with spring constant.

Unit 8: Fluids

Question 488

Problem: Find the gauge pressure at a depth of 2 m in water. Use rho = 1000 kg/m^3.

Direct Answer: 19600 Pa

Detailed Work: Gauge pressure in a fluid is P = rho g h. Substitute: P = 1000(9.8)(2) = 19600 Pa.

Memory Key: Fluid pressure increases linearly with depth.

Unit 1: Kinematics

Question 489

Problem: A ball rolls horizontally off a table at 10 m/s from a height of 11 m. Ignoring air resistance, estimate the time to hit the ground and horizontal range.

Direct Answer: t = 1.498 s, range = 14.983 m

Detailed Work: Vertical motion determines time: h = $0.5gt^2$, so t = √(2h/g) = √(2(11)/9.8) = 1.498 s. Horizontal speed is constant, so range = vx*t = 10(1.498) = 14.983 m.

Memory Key: Projectile motion separates horizontal and vertical motion.

Unit 2: Force and Translational Dynamics

Question 490

Problem: A 11 kg box slides on a level surface with kinetic friction coefficient 0.25. Find the friction force magnitude.

Direct Answer: 26.95 N

Detailed Work: On a level surface, normal force N = mg = 11(9.8). Kinetic friction is f_k = mu_k N = 0.25(11)(9.8) = 26.95 N.

Memory Key: On level ground, friction magnitude is mu mg.

Unit 3: Work, Energy, and Power

Question 491

Problem: A 4 kg object is lifted 9 m. Find the gain in gravitational potential energy.

Direct Answer: 352.8 J

Detailed Work: Gravitational potential energy change is delta U = mgh = 4(9.8)(9) = 352.8 J.

Memory Key: Near Earth's surface, gravitational potential energy is mgh.

Unit 4: Linear Momentum

Question 492

Problem: A 3 kg cart moving at 9 m/s sticks to a 4 kg cart at rest. Find their final speed.

Direct Answer: 3.857 m/s

Detailed Work: Momentum is conserved in a perfectly inelastic collision. Initial momentum = 3(9) + 4(0) = 27. Total mass = 7. Final speed $v_f = 27/7 = 3.857$ m/s.

Memory Key: For sticking collisions, use total momentum divided by total mass.

Unit 5: Torque and Rotational Dynamics

Question 493

Problem: A net torque of 4 N*m acts on an object with rotational inertia 3 kg*m². Find the angular acceleration.

Direct Answer: 1.333 rad/s²

Detailed Work: Rotational Newton's second law is tau_net = I alpha. Solve alpha = tau/I = 4/3 = 1.333 rad/s².

Memory Key: Rotational analog: torque equals rotational inertia times angular acceleration.

Unit 6: Energy and Momentum of Rotating Systems

Question 494

Problem: Find the angular momentum of a rigid body with rotational inertia 8 kg*m² and angular speed 9 rad/s.

Direct Answer: 72 kg*m²/s

Detailed Work: For a rigid body rotating about a fixed axis, angular momentum is L = I omega. Thus L=8(9)=72 kg*m²/s.

Memory Key: Angular momentum is I times omega.

Unit 7: Oscillations

Question 495

Problem: A simple pendulum has length 3.5 m. Estimate its period for small oscillations.

Direct Answer: 3.755 s

Detailed Work: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. Substitute L=3.5 m and g=9.8 m/s²: T = 3.755 s.

Memory Key: Pendulum period depends on length, not mass, for small oscillations.

Unit 8: Fluids

Question 496

Problem: An object displaces 0.009 m³ of water. Find the buoyant force.

Direct Answer: 88.2 N

Detailed Work: Buoyant force equals the weight of displaced fluid: F_b = rho g V = 1000(9.8)(0.009) = 88.2 N.

Memory Key: Buoyant force is the weight of the displaced fluid.

Unit 1: Kinematics

Question 497

Problem: A cart starts with velocity 8 m/s and accelerates at 4 m/s² for 5 s. Find its final velocity and displacement.

Direct Answer: v = 28 m/s, displacement = 90 m

Detailed Work: Use constant-acceleration equations. Final velocity: v = v0 + at = 8+4(5) = 28 m/s. Displacement: delta x = v0t + (½)at² = 8(5) + 0.5(4)(5)² = 90 m.

Memory Key: For constant acceleration, use v=v0+at and delta x=v0t+0.5at².

Unit 2: Force and Translational Dynamics

Question 498

Problem: A force of 26 N pulls a 10 kg object at 30 degrees above the horizontal. Ignoring friction, find the horizontal acceleration.

Direct Answer: 2.252 m/s²

Detailed Work: Only the horizontal component accelerates the object: F_x = F cos 30 = 26(0.866). Then a = F_x/m = 22.516/10 = 2.252 m/s².

Memory Key: Resolve angled forces into components before applying F=ma.

Unit 3: Work, Energy, and Power

Question 499

Problem: A constant horizontal force of 13 N moves a box 9 m in the same direction. Find the work done.

Direct Answer: 117 J

Detailed Work: Work is W = Fd cos theta. Since force and displacement are in the same direction, theta=0 and cos 0=1. W=13(9)=117 J.

Memory Key: Work uses only the component of force along displacement.

Unit 4: Linear Momentum

Question 500

Problem: Find the momentum of a 8 kg object moving at 5 m/s.

Direct Answer: 40 kg*m/s

Detailed Work: Momentum is p = mv. Substitute m=8 kg and v=5 m/s: p=8(5)=40 kg*m/s.

Memory Key: Momentum is mass times velocity.